TWG "Mathematical potential, creativity and talent"

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1. INTRODUCTION

The purpose of the Thematic Working Group (TWG) "Mathematical potential, creativity and talent" was to raise the attention of mathematics educators and researchers to the field of mathematical potential, creativity and talent. The group activities were directed at promoting discussions of the theoretical foundations of mathematical creativity and talent, encouraging in-depth empirical studies on these topics and offering a forum for presentation and discussion of mathematical activities, which develop mathematical creativity and promote mathematical talent. The group was initiated in 2011 as a continuation of the work of the TWG *Advanced* Mathematical Thinking (AMT).

The AMT group included participants from two major trends (in accordance with the distinction made by Tall 1991): The first trend was associated with AMT as focusing on learning and understanding of the university mathematic, (AMTE at an *absolute level* in Tall's terms). For example, such studies as "Secondary-tertiary transition and students' difficulties: the example of duality" by Martine De Vleeschouwer (2011) or "Conceptual change and connections in analysis" by Kristina Juter (2011).

The second trend in the AMT group was associated with high achievements in school mathematics, creative mathematical performance, and solving non-standard / Olympiad problems, a (at *relevant level* in Tall's terms). These studies explored creativity, potential and talent in school mathematics. Study by Voica and Peltzer (2011) who compared problem posing by novice and experts is an example of the study presented at TWG AMT in the relative direction.

The issues of creativity, potential, and talent were less relevant to the participants from the first sub-group whereas researchers from the second sub-group were less connected to the issues discussed with respect to mathematics studied at University Level. Thus it was suggested by Roza Leikin to split the group AMT into two TWG: University mathematics and TWG: Mathematical potential, creativity and talent.

Note that the terms mathematical giftedness, mathematical potential, high mathematical abilities and mathematical talent are used often interchangeably. However, mathematical giftedness implies high mathematical abilities while giftedness is perceived as an inborn personal characteristic, in contrast with high abilities which is a dynamic characteristic that can be developed (Leikin, 2014). In this TWG mathematical giftedness was connected to the high level performance in mathematics and with mathematical creativity.

Let us clarify to the reader that in this chapter we do not adopt a single definition for talent, giftedness, or exceptional mathematical abilities. The reason is that some researchers in the TWG used these terms interchangeably whereas other researchers selected one of these terms for a specific context and a specific reason. For instance, some researchers chose to use the term "promising students" instead of "gifted" for students of young age who carried certain characteristics but did not have any official accomplishment or records which would grant them the title "gifted".

The TWG "Mathematical potential, creativity and talent" encouraged international participation and exchange of ideas on the topics of mathematical creativity, potential and talent and varied in the research paradigms, basic theories, and research methodologies. The focus of the research studies also varied from individual students and classrooms, teachers and teacher education as well as experiences of research mathematicians. During the three meetings (2011, 2013, 2015) of this TWG, 44 research studies and 7 posters were discussed; and approximately 70 researchers from 15 countries participated. In particular the participants' discussions addressed the following overarching themes:

(a) Theoretical underpinnings of the concepts of mathematical talent and creativity (definitions, origins and characteristics).

(b) Methodological approaches and tools that are useful for identifying mathematically talented and creative students;

(c) The nature and features of mathematical approaches, tasks and activities that are challenging, fundamentally free of routine, inquiry-based, and rich in authentic mathematical problem solving for the development of mathematical creativity and realization of mathematical talent;

(d) The relationship between mathematical talent, motivation, effort and mathematical creativity;

(e) Teacher training associated with teaching gifted students as well as promoting mathematical creativity in all students.

(f) Historical and sociological analysis of issues relevant to mathematical creativity and giftedness.

This chapter addresses these issues in the following three sections: (a) Mathematical creativity, (b) Mathematical talent/potential/giftedness, and (c) Methodological approaches used in studies associated with these topics. In these sections we attempt to bring together some of the findings and open issues of the research studies discussed during the CERME meetings.

2. MATHEMATICAL CREATIVITY

2.1 Theoretical foundations and empirical studies of mathematical creativity and its development

Discussions on the theoretical foundations of creativity attended definitions and origins of creativity, while empirical studies concentrated on how to identify it and measure it. During the meetings, no consensus was reached regarding a single definition of mathematical creativity. The numerous studies used different definitions for creativity (Kruteskii, 1976; Polya, 1973; Torrance, 1974), different methodologies (qualitative and quantitative), addressed different populations (preschool, primary, secondary and tertiary level as well as prospective teachers), various mathematical topics (equations, fractions, proofs in geometry, patterns) and assessed creativity based on cognitive and /or social characteristics. One thing that researchers appeared to be in agreement is that the definition of creativity depended on the context under examination.

Amongst the various theoretical backgrounds that were used for the measurement of mathematical creativity, Torrance's one was the most dominant. Torrance (1974) suggested that to evaluate one's creativity four components need to be measured: fluency, flexibility, elaboration and originality. In mathematics education, the measurement of the three components, namely fluency, flexibility and originality have been most often adopted. Difficulties have been found in assessing elaboration in mathematics. Leikin (2009) suggested that multiple solution tasks is a good means to mathematical creativity and also introduced a model for evaluating mathematical creativity which takes in consideration students' fluency, flexibility and originality. Leikin and Kloss (2011) showed that while 8th and 10th grade students' success in problem solving is highly correlated with fluency and flexibility, originality appeared to be a special mental quality. Interestingly, mathematics teaching did not seem to advance students' creativity. Leikin, Levav-Waynberg and Guberman (2011), through two comparative studies investigating prospective mathematics teachers, found that although fluency and flexibility significantly increased, the same did not occur for originality. Thus, they concluded that fluency and flexibility are of dynamic nature whereas originality is a "gift" and also the strongest component out of the three in determining creativity. However, not all researchers agreed that mental flexibility is related to the creative process (Czarnocha, 2015). Czarnocha, (2015) argued that Koestler's ideas of "bisociation and simultaneity of attention" could serve as a theoretical framework for investigating creativity.

A number of studies focused on the relation between general creativity and mathematical creativity. Kattou, Christou, and Pitta-Pantazi (2015) suggested that general creativity and mathematical creativity are distinct and one cannot predict mathematical creativity based on general creativity and vice versa.

Levenson (2011), explored notion of collective creativity and examined the way in which fluency, flexibility and originality can be used to describe collective creativity. She also examined the relation between individual and collective mathematical creative both in respect of process and product. She suggested that the attempts to promote collective creativity could possibly also promote individual creativity.

Although empirical studies on mathematical creativity were conducted in all levels of education preschool (Münz, 2013), primary (Kattou et al., 2015, Levenson, 2011), secondary (Bureš & Nováková, 2015; Leikin & Kloss, 2011) and tertiary levels (Els De Geest, 2013; Karakok, Savic, Tang & El Turkey, 2015), Münz (2013), stressed that research on mathematical creativity in early childhood is not taken too much in account and is not clear how mathematical creativity is expressed and how it may be observed in these young ages. On the other end of the educational ladder, Karakok, et al. (2015) investigated mathematicians' views on undergraduate students' creativity. Originality and aesthetics, appeared to capture undergraduate students' view of creativity. In addition to this, they found that all participants believed that creativity could be encouraged in undergraduate courses and provided suggestions on how to cultivate creativity in courses focusing on proving and problem solving.

2.2 Mathematical activities aimed at developing mathematical creativity

A number of researchers who investigated the development of students' creativity, explored students' mathematical investigations and problem solving culture via heuristic strategies (Bureš & Nováková, 2015), the impact of high versus low guidance structured tasks on the development of creativity (Palha, Schuiterma, van Boxtel, & Peetsma, 2015). Some researchers suggested problem posing as a means for developing students creativity (Singer, Pelczer, & Voica, 2011; 2015), and others investigated the use of technology (Sophocleous & Pitta-Pantazi, 2011). Regarding the impact that the guidance in mathematical tasks, has on the development of creativity, Palha et al. (2015) did not find any significant differences in students' fluency and flexibility. On the other hand Safuanov, Atanasyan and Ovsyannikova (2015) claimed that open-ended exploratory learning is an effective tool for the development of creativity. Singer et al. (2015) found that students fail to pose mathematically consistent problems when they must rely on deep structures of mathematical concepts and strategies.

They also found that the abstraction level of the solution process appeared to determine the novelty of the newly posed problems and seemed to be a good predictor of the students' creative potential (Singer et al, 2011). Regarding the impact of technology on the development of mathematical creativity Sophocleous and Pitta-Pantazi (2011), found that students' engagement with an interactive 3D geometry software improved their creative abilities due to the opportunities it offered to imagine, synthesize and elaborate.

Among the studies which explored the development of teachers' creativity was Maj's (2011), which investigated whether mathematics teachers' creativity can be developed through a series of workshops. She found that conscious didactical methods and tools are needed to develop creative mathematical activities and it is not something that can be spontaneously developed. Leikin and Elgrabli (2015) while investigating prospective teachers and an expert in problem solving, reached the conclusion that discovery skills can be developed in people with different levels of problem solving expertise while the range of this development depends on the expertise.

2.3 Teaching for mathematical creativity

Fewer studies explored whole courses or practices, rather than activities, for the development of mathematical creativity. De Geest (2013) presented an undergraduate distance learning course for mathematics education students which aimed and developed students' thinking through the "possibility thinking" framework. He found that the pedagogical approaches suggested by this framework offered opportunities to students to: "play" and explore, ask "what if" questions, take risks and not have the "fear of failure", be more imaginative, make connections with other areas of mathematics, and be aware of their own learning. Monstand (2015), suggested that: conjecturing, problem posing and rule negotiation are among the essential mathematical practices for acting creatively in mathematics.

Furthermore, Sarrazy and Novotná (2013) investigated the effects of teaching mathematical creativity in heterogeneous groups and especially the impact on highly able students. They claimed that teaching is efficient if teachers take in consideration the zone of proximal development and that grouping pupils according to their level does not bring optimization of results.

Finally, Émin, Essonnier, Filho, Mercat, and Trgalova, (2015) investigated the reactions of students when they used electronic books to work on mathematical creativity tasks. Interestingly enough they found that high achievers, gave the answers that they felt their teachers wanted but were not willing to do anything further than this because it would exceed their obligations as students.

2.4 Teacher preparation for developing mathematical creativity

Studies that investigated prospective teachers and mathematical creativity evolved in different directions. Some investigated pre-service teachers' mathematical creativity (Birkeland, 2015) while others teachers' perception of creativity stimulating activities (Desli & Zioga, 2015; Sinitsky, 2015). Based on Lithner's distinction between imitative and creative reasoning, Birkeland, (2015) suggested that in some cases pre-service teachers' reasoning was neither imitative nor creative. Desli and Zioga (2015) explored the features that pre-service and in-service teachers consider appropriate for tasks that promote creativity and how they envision creativity in school. They found that although both groups of teachers identified general characteristics of mathematical creativity when asked to present tasks, these characteristics were not present in the tasks that they chose. Furthermore, pre-service teachers related mathematical creativity to the use of interesting stories in the mathematics classrooms, students' own constructions, use of materials and non-algorithmic thinking. While, in-service teachers related creativity mainly with students' problem posing activities. Similarly, Sinitsky (2015), investigated pre-service teachers' perceptions of creativity and found that pre-service teachers believed that elementary school mathematics are not suitable for promoting creativity. Vale and Primentel (2011) presented teachers with mathematical pattern tasks and suggested that these can be used to develop students' creativity as well as their higher order thinking. Overall, researchers seem to agree that there is a need to educate teachers about mathematical creativity.

3. MATHEMATICAL TALENT/GIFTEDNESS/POTENTIAL:

3.1 Theoretical foundations and empirical studies on mathematical talent/giftedness/potential and its development

A topic that has been of great interest to researchers of the TWG, was the identification of mathematically talented students. Various theoretical backgrounds (Greenes, 1981; Kießwetter, 1992; Kruteskii, 1976; Renzulli, 1978) and methodological approaches (qualitative, quantitative, mixed) have been used for the identification of mathematically gifted. Even though there is still no clear consensus on who may be named mathematically talented, gifted or potentially gifted, researchers seem to agree that multiple sources of information are needed to identify them.

Articles presented, tried to empirically identify mathematically gifted students. Brandl (2011) argued that high attainment in mathematics does not necessarily mean mathematically giftedness and not all mathematically gifted students have high attainment in mathematics. He also claimed that creativity, curiosity, out-of-the-box thinking, flexibility, and noncompliant are characteristics of mathematically gifted students.

Problem solving had a central role in the majority of studies that explored how mathematical gifted students may be identified. Rott (2015), found that gifted students were significantly more successful in problem solving than regular students. However, gifted students used significantly fewer heuristics compared to equally successful novices possibly because regular students, used heuristics to compensate for the lack of mental flexibility.

Another topic that had a central role was the mutual relation between mathematical giftedness was the topic of mathematical creativity. To date no consensus has been reached about the relation between the two. Leikin and her colleagues (Lev & Leikin, 2013) studied intensively the relation between mathematical creativity, general giftedness and mathematical excellence. They found that that general giftedness and excellence in mathematics has a major effect on students' creativity. Furthermore, they found that students excelling in mathematics who are gifted are significantly more flexible than non-gifted students and argued that originality is related to general giftedness. Kattou, et al. (2011) and Kontoyianni, Kattou, Pitta-Pantazi and Christou (2011), findings are in accord to the above results and argued that mathematical giftedness may be described in terms of mathematical ability and mathematical creativity. They also claimed that mathematical ability can be predicted by mathematical creativity.

Studies presented in the TWG were not limited to students' cognitive characteristics which may be observed through students' external behavior, responses or interviews. A number of studies conducted by Leikin and her colleagues, were interdisciplinary and multidimensional and aimed towards deeper understanding of the nature of mathematical giftedness based on a combination of theories in mathematics education and general giftedness through application of, cognitive, and neurocognitive research methodologies. This research group distinguished between general giftedness and excellence in school mathematics. In one of their studies Baruch-Paz, Leikin, and Leikin (2013), investigated the differences in memory and speed of processing in general gifted students and those students that excel in mathematics. They found that mathematical giftedness can be defined as a combination of general giftedness and excellence in mathematics and that there are interrelated but qualitatively different phenomena in mathematical gifted students. Specifically they found that memory and speed of processing abilities seem to be important factors in explaining mathematical giftedness. They also found that there are differences in speed of processing which are task depended (Baruch-Paz, Leikin, & Leikin, 2015). Szabo (2015) also found that memory has a critical role to play in the students' choice of problem solving method. Waisman, Leikin, Shaul and Leikin (2013), when examining brain potentials of gifted students and students who excel in mathematics, found that giftedness was expressed in more efficient brain functioning. Finally, Kontoyianni et al. (2011) claimed that fluid intelligence and self-perceptions could predict mathematical giftedness.

A number of researchers were interested in the motivational aspects that may allow the identification of mathematically gifted students. Benölken (2015) investigated students' interest and attitudes towards mathematics. He found that motivational factors were critical for the identification of mathematical talent and that among the students who were mathematically talented boys had a stronger interest in mathematics.

Furthermore, researchers investigated the identification of twice exceptional children, and specifically mathematically gifted with developmental impairment. Nordheimer and Brandl (2015), suggested the combination of written tests and process-based analyses of lessons. Nolte (2013) claimed that high abilities allow hiding developmental disorders and impairments in regular classrooms, and that developmental disorders can hide high cognitive abilities with the risk of losing these talents. All researchers, suggested that we need balanced programs which will support these students' talents and offer intervention for their disabilities, so that these talents do not get lost.

3.2 Mathematical activities aimed at promoting mathematical talent/giftedness/potential

Another important terrain covered was mathematical activities promoting mathematical talent. Some researchers, offered models for analyzing the typologies of problems (Ivanov, Ivanova, & Stolboy, 2013), others investigated the activities in textbooks for mathematically gifted (Karp, 2013), while others suggested approaches (Aizikovitsh-Udi & Amit, 2011; Pitta-Pantazi, Christou, Kattou, Sophocleous, & Pittalis, 2015; Sarrazy & Novotná, 2011; Schindler & Joklitschke, 2015) or principles (Kasuba, 2013) that should be used to facilitate mathematical talent.

Ivanov, et al. (2013) suggested a two-dimensional model to describe the typologies of mathematical problems based on the kinds of activities that students use in problem-solving and the schemes of reasoning they apply. They suggested that their analysis offers the possibility to identify similarities amongst problems and helps educators to develop new problems.

Karp (2013) investigated the differences between sets of problems from textbooks and from "ordinary" textbooks for the mathematically gifted. He claimed that textbook authors for mathematically gifted, seem to believe that gifted students learn algorithms more easily and are able to transfer and apply mathematical ideas in other domains and contexts more quickly than ordinary students.

The principles which should guide the writing of mathematical problems and organization of working sessions for mathematically gifted also attracted researchers' attention. For instance, Kasuba (2013) analysed the practice of writing mathematic problems. He offered a list of principles that may guide this processes: (a) prior knowledge, (b) clarity, (b) attractiveness, (c) intellectual freedom, (d) chance to act as mathematicians, (e) age appropriateness, (f) supportive environment.

Other researchers suggested or investigated specific approaches which may enhance mathematical talent. Sarrazy and Novotná (2011) investigated the impact of combining didactical and modelling of mathematical competences. Aizikovitsh-Udi and Amit (2011) suggested that the infusion approach may promote students' critical thinking. Schindler and Joklitschke (2015) explored students' approaches and abilities, especially mathematization and generalization. They suggested that giving adequate cues to focus students' attention on specific aspect of a graph, or asking them more explicitly to consciously reflect about the similarities of problem situations could optimize the tasks and support students. Pitta-Pantazi, et al. (2015) suggested that students and especially talented ones should be challenged with problems that combine a collection of mathematical competences (i.e., digital, communication in mother tongue, learning to learn and sense of initiative).

3.3 Teaching mathematically talented students

When discussing didactical principles of working with talented students participants made distinctions between affective, social and cognitive domains. Among others, the following issues were found extremely important:

When designing programs for talented students mathematics educators have to consider the social consequences of treating able children differently. If children are offered the same curriculum as everyone else, there is no point in separating them in different groups. Ability grouping requires offering a qualitatively different curriculum. Furthermore, interdisciplinary and multidisciplinary nature of the curriculum are important principles in the education of talented since spending majority of time on mathematics can sometimes stop them from doing well in other subjects which they may excel at. Additionaly, any acceleration or enrichment program should suit the individual differences of these students not only cognitively but in the affective field as well.

Moreover, regarding classroom/school culture for talented students it was expressed that these students need to have well trained teachers and an environment where ability is celebrated. The role and the characteristics of teachers who teach mathematically gifted students were also of concern to researchers. Karp and Busev (2015) claimed that teaching highly gifted students requires teachers' creativity and exemplified their ideas with the two cases of prominent Russian mathematicians. In addition, Pelczer, Singer and Voica (2013) studied the challenges and limitations faced by a mathematics teachers when teaching talented students in mixed classrooms and found mismatches between teachers' self-efficacy beliefs and specialised content knowledge.

4. METHODOLOGIES USED IN THESE STUDIES

The contributions in this TWG varied in the research methodologies:

- Theoretical investigation of the concepts of mathematical talent and creativity.
- Empirical research on the issues underlined above.
- Historical and sociological analysis of the relevant issues on mathematical creativity and giftedness.
- Presentation of mathematically challenging tasks

Twenty four studies were quantitative, fifteen were qualitative, two studies used mixed methodology and three studies were theoretical. In the quantitative studies the majority of the researchers used tests whereas most of the qualitative studies used a mixture of methods and tools (interviews, tests, questionnaires, observations or various artifacts such as textbooks, students' work or notes). One of the innovations regarding the methodological approaches presented in this group was also the inclusion of neuropsychological approaches by Leikin and her colleagues (Waisman, Leikin, Shaul, & Leikin, 2013) investigating talented students and mathematical creativity.

However, there are still some methodological limitations that further research could address in the years to come. Primarily, we need to better establish the validity and reliability of instruments designed to capture mathematical creativity and giftedness. At the same time, developing more economical and conducive tools for assessing these concepts could better portray these two concepts. It is also critical to conduct more studies that integrate both quantitative and qualitative methods, in order to offset the methodological problems of each tradition and gain a wider and deeper understanding of mathematical creativity and talent. Finally, another methodological issue pertains to meta-analyses, which will enable us to draw more causal inferences about mathematical creativity and giftedness.

5. SUMMARY

The Thematic Working Group Mathematical Potential, Creativity and Talent encouraged discussions between research mathematicians, mathematics educators and educational researchers. Still many questions remain open, either because they were left unattained or because they raised disagreement among the group participants. Some of these questions are the following:

- The studies presented at the WG differed in theoretical models and frameworks used. The frameworks determine research procedures and tools and more-over the study results that we described above in this chapter. It would be of interest to identify whether different models and frameworks are compatible and whether these combinations can result in deepening our understanding of the phenomena of mathematical potential and creativity.
- Not less important the variety of research approaches used by the TWG participants led to the question "What is a reasonable balance between qualitative and quantitative studies in the field of giftedness and creativity that can lead to a better understanding of these phenomena? What other

research methodologies may be facilitating and supporting the investigation of mathematical creativity and talent?

- As we presented above researchers in the TWG used different definitions of mathematical potential and mathematical creativity in their studies. These definitions were often incompatible and lead to debate and disagreement between researchers. Based on these observations we argue that further work should be dome in order to establish common ground and shared language in the field.
- There was obvious interest among participants in the approaches of nurturing mathematical potential and creativity. From reading papers published in post-conference proceedings of 2011, 2013, 2015 the readers can borrow various ideas about educational approaches, mathematical tasks and activities that may be beneficial for the promotion of mathematical talent. We suggest that further systematic research on the effectiveness of these approaches is needed. Such studies - comparative or descriptive, qualitative and quantitative – should help mathematics educators to perform knowledgeable choices of the tools for promotion of students' creativity and giftedness in mathematics.
- Many researchers discussed mathematical creativity as one of the important components in teacher professional development. Still, teachers' competences related to the development of students mathematical potential and creativity are not clearly identified. Correspondingly future research can search for answers to the questions: What should be the characteristics of teachers and what kind of teachers' education programs are needed to promote mathematical creativity and talent? What advancements have been made internationally so far?

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