14 Mathematics education and language:

Lessons and directions from two decades of research

Núria Planas, Autonomous University of Barcelona, Spain
Candia Morgan, University College London Institute of Education, UK
Marcus Schütte, Technical University of Dresden, Germany

This is a draft document. It is inappropriate to cite it as other than work in progress.

1. Introduction

Several decades after the conceptualization of language as a system of signs provided by the structuralist linguistic paradigm in language research (Saussure, 1968), a range of questions about language and ways of tackling them have been developed inside and outside the field of mathematics education. In the middle of a diversity of premises about conceptions of language, the expansion of the linguistic paradigm started in other social sciences (Gumperz & Levinson, 1996). Today we know that language is a corpus of linguistic rules and texts, but also and importantly an array of realities and contexts of use for rules and texts. In this Chapter, we will argue that the progress of mathematics education and language research is taking place through a complex expansion of the linguistic paradigm rather than an overthrow, with an increase in the scope of the domain and in the spread of cultural and social claims. The questions addressed will be:

- What is the scope of the research on mathematics education and language?
- What are the newer approaches in the ERME contemporary domain of language and mathematics education research?
- How can we map the emergence and development of these approaches to the development of some classical lines of interest in the domain?

Studying the process of expansion of our knowledge of language in mathematics education research, along the two decades of the European Society for Research in Mathematics Education (ERME), has led us to identify two broad related approaches: *language as culture* and *language as discourse*. In their contemporary forms, these approaches are not mutually exclusive alternatives but complementary directions in the renewed interpretations of language as a system of not only linguistic signs. While the first approach understands human activity as the result of cultural symbolic codes and

structures, the second one establishes its basis in the orderliness of the social world. These approaches can be compared in terms of the periods during which each has matured, and can be traced back into views of language as a window on systems of interaction:

> "The quick evaporation and the situative uniqueness of verbal accomplishments impede the reflection on such interactive procedures ... "Creative and productive writing" in such a sense is not only a category of native language classes, but in general a platform for reflection on classroom related processes of symbolization." (Krummheuer, 1998, p. 339)

The postulation of the dependence of mathematical language on cultural contexts and the problematization of mathematics as a universal language preceded the foundation of ERME. Recognition of the fact that speakers of mathematics speak this "language" differently in different cultures or groups began to gain traction in the context of the increasing influence of the ethnomathematical stance (D'Ambrosio 1985). At that time, the turn to culture was being experienced in the framing of mathematics classrooms as cultures (Cobb & Bauersfeld, 1995) and of (school) mathematics as product of culture (Bishop, 1988). Some years later, the turn to the social (Lerman, 2000) would enter ERME:

> "What we understand by mathematics is far from being a unified body of knowledge determined by the practices of professional mathematicians, but rather a series of "knowledges" and "language games" bounded to a diversity of practices, all of which have a family resemblance … If mathematics-related language games are present in many spheres of practice, the meaning of them is also constituted in relation to those practices and their discursive elements." (Valero, 2009, pp. 14-15)

The persistence, articulation and renewal of approaches recommend characterising the development of the domain as *a continuum of complexity*. While moving to language as culture has added the complexity of communities of practice and activity, moving to language as discourse has added the complexity of social processes. This goes with the complexity added by views of language as integral to the understanding of some classical research topics like student learning, classroom practice or teacher education. This is in line with the argument that the turn to language is a contemporary context in the field with a role in broadening the scope of research (Morgan, 2013).

In what follows, in Section 2 we discuss what is involved in international research on mathematics education and language in order to provide some clarity regarding the standpoints that have guided our review. In Section 3 we survey research reported at CERME since 1998 as a benchmark for assessing the progress in the domain. Our survey

work uncovers some overlapping lines of interest, which appear as common themes across papers presented in the Working Group on Language and Mathematics (WG). These lines complement each other in ways that open up the articulation of approaches and domains in the field. Finally, the discussion of the complexity of research on mathematics education and language leads to map some gaps and future directions in Section 4.

2. What does it mean (to) research on mathematics and language?

Within the field of mathematics education research, language has long been seen to be an important focus. The seminal 1979 review article by Austin and Howson cited research into mathematics and language dating back to the 1940s, with a substantial body of research beginning to establish itself in the early 1970s. Nearly 40 years later, the research on mathematics education and language developed within ERME and beyond still addresses the broad themes identified by these authors: the language of the learner (i.e. the language or languages and linguistic skills brought to the mathematics classroom by learners), the language of the teacher (and the classroom), and the language of mathematics. However, we can see substantial development in the sophistication with which these themes are conceptualized and addressed. We see more widespread and systematic engagement with and utilization of theories of language and communication developed in other social scientific fields, including psychology, sociology, philosophy, linguistics, semiotics, and anthropology, as well as the development of theoretical perspectives addressing the role of language in mathematics education. There has also been a growth in the diversity and complexity of the domain as researchers draw on a wider range of theoretical resources and combine these in new ways.

One source of diversity is the fundamental fact that research on mathematics and language includes both that which takes language itself as the object of study and that which uses language as a vehicle for studying other phenomena. Both types of research formulate descriptions of language in use in a mathematical context but analyse and interpret the resulting descriptions in different ways. The description of language may draw on tools from linguistics but these do not always serve the purpose of distinguishing characteristics of language use that are of interest to mathematics education. A major contribution was made by the publication of Pimm's (1987) book "Speaking mathematically: Communication in the mathematics classroom", widely cited as a means of characterising language and its use in mathematics educational contexts. Nevertheless, the introduction to

the WG papers at CERME4 identified a need to develop greater rigour in the ways in which we define and distinguish between mathematical and "everyday" language:

> A basic goal for research is to understand what is specific in mathematical language, going beyond both the wrong ideas that the core of mathematical meaning (or, worse, mathematical truth) is embedded into symbolic expressions and that mathematical language has no specific features relevant from the viewpoint of education. (Morgan, Ferrari, Duval, & Johnsen Høines, 2005, p. 791)

The significance of this goal appears even stronger in the light of the development of theoretical understanding of mathematics itself as an essentially discursive activity. Developing understanding of the distinctive nature of mathematical communication is a necessary element of any study of mathematical activity. This is the case whether one adopts the 'strong' discursive position advocated by Sfard (2008), who argues that mathematical objects have no existence independent of the discursive means of communicating about them, or a less absolutist position such as that stated by Duval (2006), who argues that we have no direct material access to mathematical objects but can only experience them through some form of 'representation' or 'realisation.' These two terms reflect two differing ontological positions: speaking of representation of a mathematical object suggests that there exists an independent object to be represented, whereas speaking of realisation proposes that the communication about an object is what gives the object existence. In any of these ontological positions, thus, mathematical activity may be perceived as the manipulation of discursive objects – or as engagement in a particular form of discourse. Understanding such activity, whether it is undertaken by professional mathematicians or in contexts of mathematics education, involves studying that discourse and understanding its characteristics.

In the field of mathematics education, understanding the specialised characteristics of mathematical language is critical as this enables us to describe and evaluate the mathematical activity of, for example, teachers and students in classrooms. Where language is the object of study, description of the language may be an end in itself, addressing the nature of "the language of mathematics". Principled description also allows us to address a wider range of questions, for example: What are the characteristics of the mathematical discourse in which students are expected to participate? How does the didactic action of the classroom induct students into (what kinds of) specialised mathematical discourse? To what extent are students engaging in specialised mathematical discourse? Indeed, from a 'strong' discursive position, any study of mathematical

knowledge and learning entails asking questions about the language used by learners and how it changes. However, the significance of language in mathematics education includes the use or function of language as well as its form. Paying attention to how language functions allows us to consider questions about processes of reasoning, argumentation and proof as well as about mathematical objects and relationships. Communication in the classroom and in other educational contexts, including curriculum, assessment and policy, also has an interpersonal function, constructing positions for students and teachers and framing relationships between them and relationships to the mathematics. Studying the interpersonal functioning of language can contribute to understanding social aspects of mathematics education, including understanding how teachers manage classroom interactions and how students from various social and cultural groups may experience mathematics education differently.

Drawing on theories that conceptualise language as constitutive, constructive or functional enables researchers to analyse what is achieved in a given context through language use, addressing the "language of the classroom" theme. The work of Heinz Steinbring and his colleagues develops and uses such a theory, specifically mathematical in its focus on the interactive generation of mathematical knowledge through classroom communication and signification (Steinbring, 2005). This international line of research has been represented at each CERME since the first. Other researchers studying language and what its use achieves have drawn on theoretical resources primarily developed in other fields, including pragmatics, social semiotics and conversation analysis. As well as studying language, its use and what it achieves, the working group has also always included contributors whose interest lies in using language as a vehicle for studying other phenomena. In particular, we see the use of linguistic data as a means of gaining insight into understanding and learning of mathematics. Especially in the first few meetings of the working group, research drawing on social constructivist and social interactionist perspectives is strongly represented, studying classroom interactions in order to observe learners' cognitive development through participation. Taking change in the forms of language as a sign of change in understanding of the mathematical concepts being discussed suggests a dualist separation of language and cognition. Some of the more recent contributions to the study of learning draw on sociocultural theories that see language and other mediating artefacts as playing a more integral role in thinking.

Although we have tend to refer to the main focus of the working group as "language", it is important to recognise that mathematical communication uses a variety of modes, of which the linguistic is only one. There are of course specialised modes, especially suited to mathematical activity, including algebraic notation, Cartesian graphs, geometric diagrams and various other symbolic and diagrammatic forms used in specific areas of mathematics. In addition to these, studies of face-to-face communication indicate the important roles that may be played by gesture in doing mathematics. The study of multimodal communication has developed substantially in recent years, stimulated in part by the transformations effected by the growth of new forms of communication technology with different affordances for communication and meaning making. As it will be shown in the next section, this development is importantly reflected in the WG, incorporating not only multimodal analyses of classroom communication but also an as yet small number of studies looking at communication mediated by technologies.

3. What have we learned from research on mathematics and language?

After examining the collection of WG papers and CERME plenaries with an emphasis on work that has added newer debates to the study of language-based research in the field, we have come to two newer approaches along with some transversal lines of interest. Since we predominantly focus on the aspects of mathematics education and language research which have been in important expansion after the founding of ERME, we do not elaborate further on language as a system. The two newer approaches are interdependent, mutually constitutive, internally complex and diverse in terms of the chronology of their consolidation (see vertical axis of Figure 1) and the complexity of their development (see horizontal axis of Figure 1). While complexity is relatively low near the origins of ERME with the primary focus on language as a system made of symbolic structures, the level of complexity becomes higher when the focus is expanded to include the cultural and historical conditions of the researched environment, and it culminates when the focus is again expanded to include the social foundation of language and mathematical activity. Along the continuum, the social becomes less subordinated to the study of culture though some cultural-based arguments remain in research that sees social order in human action dependent on and not interpretable outside of shared cultural structures.

Our characterization of the progress of the domain also applied to the characterization of internal progress within the newer approaches (see Figures 2 and 3). Both of them involve some continuum of complexity relative to the structural, linguistic, cultural and social

components addressed over time. On the other hand, the various related lines of interest that will be next summarized are not defined to coincide with research programs of individual authors or concrete groups. Our choice of a few authors does not acknowledge the valuable contributions made by all authors and groups with participation in our Society.

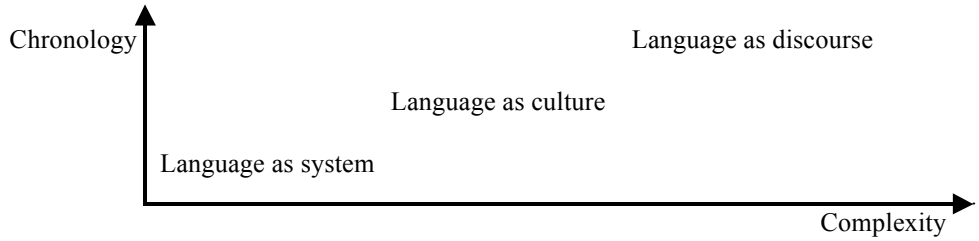

Figure 1. Representation of the expansion of the ERME domain

2.1. Language as culture in ERME

The views of language as a window into particular cultures and communities of mathematical practice have internationally evolved in combination with views of language as a tool for cultural and historical analyses of specific cultural configurations like the mathematics classroom (Radford, 2003). In ERME, we do not find a unified cultural approach to language either, but a set of distinct ways of analyzing and theorizing about language in the field with the key standpoints of language as cultural resource and language performance as cultural practice. The social reasons for the forms, meanings and functions of language are considered in relation to the network of cultural and historical conventions surrounding them. All forms, meanings and functions therefore lie in what they linguistically designate as the basis for what they culturally connote. Implicit in this line of argument is the idea that the foundation of mathematical activity is cultural.

Our survey work has led us to identify three major lines of interest within this approach: *cross-cultural studies*, *classroom (mathematical) talk* and *cultural semiotics*. Each line is the historical result of challenges posed to earlier practices of research in which either the (mathematical) culture of the mathematics classroom or the culture of (school) mathematics were in focus. In Figure 2, the development of the newer cultural approach in the domain is represented in terms of a continuum of complexity, similarly to the representation of the expansion of the overall domain (Figure 1). Despite the continuum is shown by means of two ends for each line, there is a diversity of practices and underlying claims involved in the many ways in which language has been studied over two decades. The line metaphor with linguistic relativity and cross-cultural studies, for example, illustrates how the claim that culture, through the structure of language, mediates the way

in which we think mathematics has evolved toward claims regarding the role of particular languages in the constitution of mathematical experience and activity. Our survey work has led to complementarily define a paired line with multilingualism (Figure 3) for research on the pervasiveness of language diversity in mathematics teaching and learning.

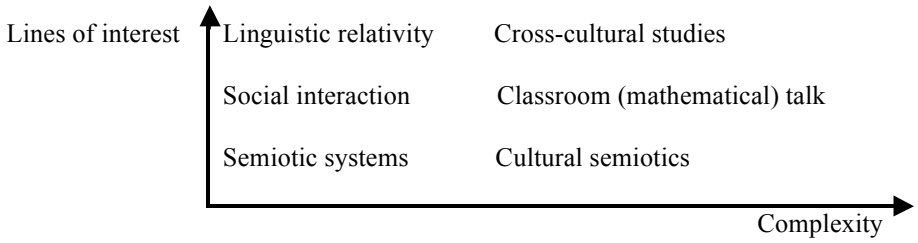

Figure 2. Representation of the expansion of language as culture

Cross-cultural studies

In recent times, linguistic relativity studies in the field have evolved from interest in how we speak about reality toward interest in how we culturally and socially construct reality (Barton, 2007). In the move forward away from the most classical formulations, the interpretation of mathematical languages and registers in relation to the singularity of specific cultures and communities has been of much relevance. Consequently, differences among languages and registers have come to be primarily seen as a representation of social interaction with and enactment of cultural practices. During the last decade of ERME, a number of ethnographically oriented papers show the influence of psycholinguistic studies and stances of linguistic relativity in the study of communicational features of language and its potential impact on the organization of mathematical activity. Nevertheless, this discussion critically goes with the recognition of the necessity to consider the cultural conditions and conventions in which the use of each language is placed.

Edmonds-Wathen (2016) examines aspects of the language of motion in Iwaidja, an indigenous Australian language, specifically the ways that the Iwaidja culture separates spatial concepts such as direction, height and movement in relation to another object. The mathematical meanings developed in the Iwaidja language for notions of directionality and movement are studied in relation to practices of orientation that are common to the Iwaidja people and compared to some of the meanings developed for these same notions in the Australian English-speaking community. Ní Ríordáin (2013) addresses cross-cultural aspects in formal contexts of mathematics teaching and learning. Syntactical and semantic differences between Irish and English are linked to differences in mathematical processing in classroom cultures with students who own these two languages. The scenario of either

Irish or English being the language of instruction is explored in relation to school cultures with institutionalized practices of transition from one language to another. Still another paper is Meaney (2007), where the two cultures refer to home and school mathematics. The achievement of the mathematics register in the early childhood is here investigated in the interaction of the child with other students and the teacher in the classroom and with members of the family at home. It is concluded that this register is differently constituted in the two contexts, with little overlap in the mathematical meanings for some mathematical notions. Lexical and grammatical mathematics registers in the cultures of home and school mathematics are discussed in relation to how patterns of authority are differently constructed by adults and communicated to the child in these two cultures.

Cultural semiotics

The development of semiotics in mathematics education research (Sáenz-Ludlow & Presmeg, 2006) has provided two contemporary lines of interest broadly known as cultural semiotics and social semiotics. The newer semiotic cultural perspectives in the field emerge from articulating psychological frames with philosophical, anthropological, historical and embodied theories of knowing and learning. Accordingly, thinking is considered "a sensuous and sign-mediated reflective activity embodied in the corporeality of actions, gestures and artifacts" (Radford, 2009, p. 36). From here a refined position capable of clarifying more nuanced explanations of the individual, the cultural and the material has been reached in line with the conceptualization of mathematics as an object of interpretation within collective meaning structures. This is the context for the semiotic cultural perspectives in the language domain, in reference to the study of how components of language carry meanings and provide support to each other for the accomplishment of mathematical knowledge, thinking, teaching and learning.

In the ERME context, various papers have adopted cultural semiotic perspectives during all these years. In Cerulli (2003), the use of mathematical software in a lesson is informed by cultural semiotics with a primary cognitive orientation in the interpretation of student access to the language of the artifact. Coppola, Mollo and Pacelli (2011) examine semantic equivalence as a component of language which provides support to the development of mathematical procedural language involved in the construction of equivalence rules in a teaching experiment. More recently, Reinhardtsen, Carlsen and Säljö (2015) interpret student learning through the study of activity around artifacts, objects, linguistic devices and signs, with an emphasis on spoken words, written text, gestures and drawing. These

three papers show a movement toward more complex multimodal analyses as a way to better capture the construction of mathematical knowledge. Embedded in the movement toward complexity, we find the question of where to place the body and the material in relation to the symbolic. Bjuland, Cestari and Borgensen (2009) relate the immediacy of the embodied experience to the multiplicity of material cultural meanings for a diagram task. Fetzer (2009) formulates the cultural status of material artefacts in mathematics teaching and learning in the sense of necessary components of the lived experiences of students and the social structure of the classroom. This work is an example of how some authors provide a critique of the reduction of materiality and social order to symbolic structures through non competing social and cultural claims.

Classroom (mathematical) talk

Starting in the German-language research, the earlier expansion of the linguistic paradigm was brought into clearer focus within the discussion of studies using interactionistic approaches of interpretive classroom research, e.g. those by Bauersfeld, Krummheuer and Voigt (1988). These studies were explicitly distanced from the previously dominant view that learning was merely an internal psychological phenomenon. Hereafter, the inclusion of interactionistic aspects of learning and teaching meant a shift of focus from the structure of objects to the structures of learning processes, and from the individual learner to the social interactions between them. The transformed understanding of learning led to the development of theories that regard meaning, thinking, and reasoning as cultural products of social activity. Krummheuer (1995) argues that learners are involved in "collective argumentations" in the learning of mathematics in primary school and it is through their increasingly autonomous participation that they learn. Based on the fundamental assumption that meaning is negotiated in interactions between several individuals and that social interaction is thus to be understood as fundamental for learning processes, language can no longer be understood only as the medium in which meaning is constructed; rather, speaking about mathematics in collective argumentations is in itself to be seen as the "doing" of mathematics and the development of meaning. Thus, language acquires a central significance, if not *the* central significance in the building of mathematical knowledge and the development of mathematical thought.

Within this understanding one can find numerous studies from the early days of ERME that focus on children's participation in classroom interaction. This focus is connected to the ostensible aim of these works to primarily understand, rather than change, learning

processes of children. Rowland (2001) for instance examines utterances of pupils and notes that language has an interactive function. It can express both social relationships and inner attitudes. He argues that linguistic means, which can be concentrated on this aspect can be used to analyze social and affective factors in mathematics teaching (see also Boero & Cosogno, 2007). Coming from this focus one can see the focal diversification onto the language design of classroom interaction. Besides focusing on understanding, recent works increasingly focus on the potential of change towards optimized terms of possibilities of learning mathematics. Dooley (2009) for example studies the teacher's role and shows how pupils pick up on each other's ideas in classroom discussion. She noted that students who make little contribution to the dialogue may nevertheless be actively engaged and pick up on others' ideas. Schütte (2009) analyses the linguistic accomplishment of instruction in a fourth-grade class. The results of these analyses grant on the one hand a hypothesis of the limited learning opportunities for a multilingual pupil body in German classes because the linguistic accomplishment of the teacher orients itself towards perceptions of unity of a monolingual 'normal' child and the diversity of the classroom is barely taken into account. With the change of focus from the learner to the language design of classroom interaction of the teacher and the interactive interdependence between all participations, special emphasis has been placed on the focus to not only describe learning processes but also show potential of change or even initiate these changes, starting in the early 2000s. Planas and Morera (2012) discuss secondary students' "revoicing" (or "retelling") and collective mathematical argumentation in different mathematical classroom situations. Their results show how student use of revoicing is able to reinforce mutual understanding. Ingram and Pitt (2015) show how mathematically capable identities, constructed in whole-class interactions, are enacted by students, which are ratified and supported by the teacher. Because of the increasing diversity of student populations, all places of learning will increasingly be characterised by a plurality of interpretations in negotiations of meaning during teaching. Therefore the goal should be to ensure teachers are sensitive to this variety of interpretations of lesson content, and to build on this by developing interpretational competences in recognising differences in interpretation.

2.2. Language as discourse in ERME

Contemporary research on language in mathematics education research has brought with it new questions about aspects of reality whose ways of manifestation may not be as explicit as naming and talking. Identity, ethnicity, multiculturalism and multilingualism (Radford

& Barwell, 2016) have brought with them new research problems and challenges. Implicit in most of this research is the standpoint that mathematical activity is socially originated and socially developed. In contrast to the cultural views of language in the domain, here the foundation of mathematical activity is explained within the pre-cultural sphere of social structures. Correspondingly, the idea of a non-culturally based social structure is fundamental in the understanding of the social constitution of language in mathematics teaching and learning. In ERME, we find various ways of conceptualizing language and mathematical activity in social structure. Some of the most recent studies address semiotic and discursive processes involved in the relationship between language and activity. In these studies, the social structure of a group in a context of practice is identified with the semiotic and discursive resources that are put in use by that group in that context.

Within this approach, we have identified three newer lines of interest in the ERME literature: *multilingualism, narrativity and discourse*, and *social semiotics*. Each line is again the historical result of challenges posed to prior practices of research. In Figure 3, the expansion of the newer social approach in the domain is represented in terms of a continuum of complexity with two ends for each line. We make our argument on the basis of progress from pre-existing areas of study in the domain (i.e. linguistic relativity, social interaction, semiotic systems) and, thus, observe this newer research in the form of evolving research. In the previous section, we have discussed work regarding cross-cultural studies, classroom (mathematical) talk and cultural semiotics (Figure 2) that can be respectively paired to work regarding the three lines in this section.

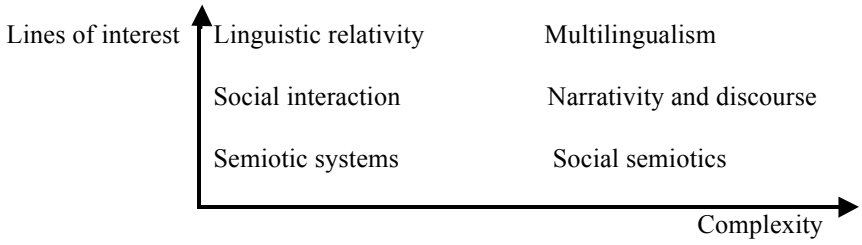

Figure 3. Representation of the expansion of language as discourse

Multilingualism

We have earlier presented the move of linguistic relativity claims in the domain toward cross-cultural studies that interpret how strong the influences between language and culture are in different communities. A complementary direction in the domain has addressed how strong the influences between language use and activity are in multilingual contexts of

mathematics teaching and learning (Barwell et al. 2016). The work by Cummins (2000) in the context of English second-language acquisition was fundamental in the initial debates of bilingual teaching and learning. In his differentiation between "basic interpersonal communicative skills" (BICS) and "cognitive academic language proficiency" (CALP), however, the positive perspective on cultural and language diversity appears to fall away, with the focus on the fact that some children learn academically relevant linguistic competences more slowly than others, or not at all. In the recent times, the perspective on children's linguistic deficits has been considerably reduced and the perspective of language as an asset rather than as a handicap has gained significant presence.

In ERME, Petrivá and Novotná (2007) made a study with German and Czech schools where a foreign language is used in mathematics teaching and learning mathematics. The main focus is on the interferences limited language proficiency can bring to student learning; despite the fact that the possibilities to eliminate the negative influence of limited language proficiency on learning are considered, there is a more deficit-oriented perspective. Since the 2000s, increasingly more studies have shown a transition from a consideration of obstacles and deficits of learners to one of resources and competences of a diverse pupil population and an impossibility of one correct mathematical discourse that needs to be achieved. Regarding this development, the paper by Schütte (2005) explains apparent gaps in communication between the teacher and students learning mathematics in their second (or third) language. The notion of multilingual habitus of teachers is used to mean that there are subconscious foundational perceptions of teachers who view monolingualism as the established and desirable standard and presuppose common social, cultural, linguistic and socioeconomic perceptions within multilingual settings, which do not apply to the majority of learners in their lessons. Also in Barwell (2015), the social dimension of language in the study of students' and teachers' language choices is addressed. This author draws on contemporary sociolinguistics of multilingualism, particularly on the notions of heteroglossia and orders of indexicality, for the analysis of a second-language primary mathematics classroom in Quebec.

Narrativity and discourse

Under the influence of narrative theories, a number of studies in the international field have contributed to developing theoretical considerations about what discourses are, how they function as well as methods of investigation about how discourses can be analysed (Planas & Valero, 2016). Here, the idea that language and textual worlds are conveyed in

relation to forms of participation, communication and being in social domains of activity is underlying the terms ‘discourse’ and ‘discourse analysis’. This idea brings with it the recognition that discourse-based research is inseparable from theories of culture, history and society as well as from questions of story representation and narrativity. The ontological stances of discourse as social action and narrativity as life experience are visible in ERME throughout various papers during the last decade. In most of them, language is conceptualized as structured through textual stories that are modified over the course of discursive practices. There is a logic that must account not only for each stage of the story being told but for each stage of the way it is being told.

Andersson (2003) examines the mathematical discourse of engineering students, while Riesbeck (2010) compares the discourses of mathematics, of everyday life and of school mathematics. In these two papers, discourse is constituted of discursive frames whose analyses on a micro level are aimed at the exploration of regularities in the related community or institution. It is taken the view that to better understand the nature of, e.g., school mathematics, it is necessary to examine how it is discursively built up in particular social contexts through specific textual productions. Jung and Schütte (2015) for example investigate to what extent the linguistic discourse in kindergarten and primary school gives children the opportunity to achieve mathematics-specific discursive competences that allow the children to participate successfully in the discourse of the mathematics classroom. Consequently, the discourse of, e.g., school mathematics becomes a historical product of human activity with room for reconstruction, change and agency. This is also highlighted in Ruthven and Hofmann (2015) in relation to the identification of two discursive frames in lesson data which “grant pupils a degree of epistemic agency” (p. 1488). Some other papers approach discourse as ways of talking and understanding by linking the term to the exploration of narratives that communicate epistemic order in classroom activity (Erath & Prediger, 2015). In Nachlieli and Tabach (2015), the narrativity in focus are the stories about personifying and the communicational routines surrounding them. In this paper, the presupposed engagement of the stories with extra-textual social conditions points to the relationship of some research in this line of interest with the spectrum of socio-cultural-political research in the field.

Social semiotics

The increased attention in the domain to the cultural and social dimensions has moved semiotic studies in the field beyond an exclusive interest in structure and system. Together

with research in the line of cultural semiotics and the Vygotskian tradition, we find important work drawing on the social production of socially organised systems of signs and meanings. In her discussion of social semiotics, Morgan (2006) argues that theories of language as social semiotic and tools of systemic functional linguistics provide some powerful ways of investigating mathematical practices and the practices of teaching and learning mathematics, as well as allowing us to develop knowledge about uses of language within mathematical practices that may be helpful for teaching and learning. Thus, a key contribution of social semiotics is its recognition of the range of functions performed by use of language and other semiotic resources. In ERME, a number of papers have adopted the standpoint of meaning making occurring in social contexts and of language use being functional within those contexts. In these papers, mathematical thinking is bound to the context and the culture in which it takes place, but this culture is ultimately explained and thought of in terms of social structure and social practice.

In their paper, Nordin and Björklund Boistrup (2015) adopt a multimodal social semiotic approach in order to capture and represent the many communicational resources such as speech, drawing and hand gestures implied in student-teacher communication when giving account of solutions at the front of the classroom. These authors conclude that by addressing multimodality it was possible to document the communication of important mathematical ideas whose reasoning had not been verbally displayed. Björklund Boistrup, and Selander (2009) propose the coordination of multimodal social semiotics and an institutional perspective in order to look at the same empirical material with different two complementary lenses with special attention to the institutional meta-function. Their research interest is assessment actions during lessons in mathematics classrooms. In Morgan and Alshwaikh (2009), different semiotic systems are conceptualised as sets of resources for the emergence of mathematical meanings during the construction of animated models using equations in a technological teaching and learning environment. The proposed multimodal analysis of an episode particularly shows ways in which mathematical and everyday speech transform the purpose of the student activity. More generally, all these papers take the assumption that semiotic activity needs to be understood in the light of institutional framings and discourses as situated in history.

4. What could we learn more in the next two decades?

We have discussed the vitality and profundity of two newer approaches and a number of lines of interest in the ERME domain of research in mathematics education and language.

There has been considerable progress in a domain with the potential to address essential research questions in the field, but several unanswered questions still remain to be addressed. To date, for instance, studies at certain age levels and with new technologies are under-represented in ERME research on language. Particularly, more research is needed in the context of how language is influenced by new technology, specifically by developing tools for analysis of multimodal discourse. New technological tools to facilitate (un-)guided collaborative work for learning mathematics enables new discourse practices (oral, chat, computer-mediated actions, gestures…) and new questions arise. For example: To what extend do the designed tools influence the classroom discourse?

Similarly, there is scarce reflection on the ways in which concerns, methods and findings from this domain can be applied to contexts of mathematics teacher education and professional development to make a difference in these areas of study and in their achievements in practice. Past research has established the connection between teachers' pedagogical knowledge and experiences of professional development. Such research could be completed with studies about the integration of teaching practices of mathematical communication and language responsiveness in the professional development agenda. Working in collaboration with mathematics teachers and mathematics teacher educators who have successfully integrated multilingual and multimodal practices in their lessons would help. With regard to multilingual practices, there is a need for further theorizing the interaction between language background and mathematics learning and teaching, as well as finding ways to support mathematics teachers. Some of the outcomes in the domain may be significant in speaking to practitioners about their ways of perceiving and using language and language diversity at the level of praxis. We propose that the domain builds upon the ground already made, by developing detailed analysis of how language can be better resourced for the purpose of mathematics teaching and learning.

There is also energy needed to critically address some practices within our own research community. In a domain where language processes are placed at the core of the agenda, the ethics and practices of power involved in the use of language by researchers remain surprisingly under-examined. This might be related to the fact that a majority of international work is conducted by researchers from a relatively small number of countries. It may also be related to the fact that English has a dominant official status within our community. Such dominance and its consequences need to be included in a more ambitious research agenda. The quality of the research experience is framed by how common and

acknowledgeable are certain languages and codes of communication. Similarly, we need to be cautious with the quality of the experience of those participating in research. A significant amount of research in the field involves researchers who do not speak the main languages of the participants in their studies. Knowing what we know now, it is unreasonable to expect that this situation does not affect research.

References

Andersson, A. (2003). The discourse of engineering students constructing concept maps in Linear Algebra. *Proceedings 3rd CERME* (p. 1).

Austin, J. L., & Howson, A. G. (1979). Language and mathematical education. *Educational Studies in Mathematics, 10*(2), 161-197.

Barton, B. (2007). *The language of mathematics: Telling mathematical tales*. New York: Springer.

Barwell, R., Clarkson, P. .... & Villavicencio, M. (Eds.) (2016). *Mathematics education and language diversity. The 21st ICMI Study*. New York: Springer.

Bauersfeld, H., Krummheuer, G., & Voigt, J. (1988). Interactional theory of learning and teaching mathematics and related microethnographical studies. In H. G. Steiner & H. Vermandel (Eds.), *Foundations and methodology of the discipline mathematics education (didactics of mathematics)* (pp. 174-188). Antwerpen: University of Antwerp.

Bishop, A. J. (1988). The interactions of mathematics education with culture. *Cultural Dynamics, 1*(2), 145-157.

Bjuland, R., Cestari, M. L., & Borgensen, H. E. (2009). A teacher's use of gesture and discourse as communicative strategies in concluding a mathematical task. *Proceedings 6th CERME* (pp. 884-893).

Björklund Boistrup, L., & Selander, S. (2009). Coordinating multimodal social semiotics and an institutional perspective in studying assessment actions in mathematics classrooms. *Proceedings 6th CERME* (pp. 1565-1574).

Boero, P., & Consigno, V. (2007). Analyzing the constructive function of natural language in classroom discussions. *Proceedings 5th CERME* (pp. 1150-1159).

Cobb, P., & Bauersfeld, H. (1995) (Eds.). *The emergence of mathematical meaning: Interaction in classroom cultures*. London: Routledge.

Coppola, C., Mollo, M., & Pacelli, T. (2011). The concept of equivalence in a socially constructed language in a primary school class. *Proceedings 7th CERME* (pp. 1302-1312).

Cerulli, M. (2003). Instruments of semiotic mediation in algebra, an example. *Proceedings 7th CERME* (pp. 1-9).

Cummins, J. (2000). *Language, power and pedagogy. Bilingual children in the crossfire*. Clevedon: Multilingual Matters.

D'Ambrosio. U. (1985). Ethnomathematics and its place in the historv and pedagogy of mathematics. *For the Learning of Mathematics, 5*(1), 44-48.

Dooley, T. (2009). A teacher's role in whole class mathematical discussion: Facilitator of performance etiquette? *Proceedings 6th CERME* (pp. 894-903).

Duval, R. (2006). A cognitive analysis of problems of comprehension in the learning of mathematics. *Educational Studies in Mathematics, 61*(1-2), 103-131.

Edmonds-Wathen, C. (2016). The grammar and conceptualisation of motion in Iwaidja. *Proceedings 9th CERME* (pp. 1361-1367).

Erath, K., & Prediger, S. (2015). Diverse epistemic participation profiles in socially established explaining practices. *Proceedings 9th CERME* (pp. 1374-1381).

Fetzer, M. (2009). Objects as participants in classroom interaction. *Proceedings 6th CERME* (pp. 974-983).

Gumperz, J. J., & Levinson, S. G. (1996) (Eds.). *Rethinking linguistic relativity*. Cambridge: Cambridge University Press.

Jewitt, C., Bezemer, J., & O'Halloran, K. (2016). *Introducing multimodality*. London: Routledge.

Jung, J., & Schütte, M. (2015). Discourses in kindergarten and how they prepare for future decontextualised learning of mathematics. *Proceedings 9th CERME* (pp. 1414-1420).

Ingram, J., & Pitt, A. (2015). Constructing mathematical competence in interaction: Whose mathematics is it. *Proceedings 9th CERME* (pp. 1407-1413).

Krummheuer, G. (1995). The Ethnography of Argumentation. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning. Interaction in classroom cultures* (pp. 229-269). Hillsdale: Lawrence Erlbaum.

Krummheuer, G. (1998). The narrative character of argumentative mathematics classroom interaction in primary education. *Proceedings 1st CERME* (pp. 331-341).

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19-44). London: Ablex.

Meaney, T. (2007). Authority relations in the acquisition of the mathematical register at home and at school. *Proceedings 5th CERME* (pp. 1230-1239).

Morgan, C. (2013). Language and mathematics: A field without boundaries. *Proceedings 8th CERME* (pp. 50-67).

Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational Studies in Mathematics, 61*, 219-245.

Morgan, C., & Alshwaikh (2009). Mathematical activity in a multisemiotic environment. *Proceedings 6th CERME* (pp. 993-1002).

Morgan, C., Ferrari, P. L., Duval, R., & Johnsen Høines, M. (2005). Language and mathematics. *Proceedings 4th CERME* (pp. 789-798).

Nachlieli, T., & Tabach, M. (2015). The discursive routine of personifying and its manifestation by two instructors. *Proceedings 9th CERME* (pp. 1447-1453).

Nordin, A. K., & Björklund Boistrup, L. (2015). Mathematical reasoning through a broad range of communicational resources. *Proceedings 9th CERME* (pp. 1461-1467).

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. London: Routledge Kegan & Paul.

Planas, N., & Morera, L. (2012). Revoicing in processes of collective mathematical argumentation among students. *Proceedings 7th CERME* (pp. 1380-1389).

Planas, N., & Valero, N. (2016). Tracing the socio-cultural-political axis in understanding mathematics education. *Second Handbook of PME* (pp. 447-479). Rotterdam: Sense.

Radford, L. (2003). On the epistemological limits of language: Mathematical knowledge and social practice during the Renaissance. *Educational Studies in Mathematics, 52*, 123-150.

Radford, L. (2009). Signs, gestures, meanings: Algebraic thinking from a cultural semiotic perspective. *Proceedings 6th CERME* (pp. 34-63).

Radford, L., & Barwell, R. (2016). Language in mathematics education research. *Second Handbook of PME* (pp. 275-313). Rotterdam: Sense.

Reinhardtsen, J., Carlsen, M., & Säljö, R. (2015). Capturing learning in classroom interaction in mathematics: Methodological considerations. *Proceedings 9th CERME* (pp. 1475-1481).

Riesbeck, E. (2010). Speaking of mathematics – Mathematics, every-day life and educational mathematics discourse. *Proceedings 6th CERME* (pp. 914-922).

Ruthven, K., & Hofmann, R. (2015). A case study of epistemic order in mathematics classroom discourse. *Proceedings 9th CERME* (pp. 1482-1489).

Rowland, T. (2001). Pragmatic perspectives on mathematics discourse. *Proceedings 2nd CERME* (pp. 408-419).

Sáenz-Ludlow, A., & Presmeg, N. (2006). Guest editorial. Semiotics perspectives on learning mathematics and communicating mathematically. *Educational Studies in Mathematics, 61*, 1-10.

Saussure, F. (1968). *Saussure ou le structuralisme sans le savoir. Présentation, choix de textes* (Edition by G. Mounin). Paris: Seghers.

Schütte, M. (2009). Linguistic accomplishment of the learning-teaching process in primary mathematics instruction. *Proceedings 6th CERME* (pp. 1023-1032).

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge: Cambridge University Press.

Steinbring, H. (2005). *The construction of new mathematical knowledge in classroom interaction: An epistemological perspective*. New York: Springer

Valero, P. (2009). Mathematics education as a network of social practices. *Proceedings 6th CERME* (pp. 54-80).