Affect and mathematical thinking: Exploring Developments, Trends, and Future Directions (Chapter 10)

Markku S. Hannula, Marilena Pantziara, and Pietro Di Martino

Abstract

This chapter reviews the research on mathematics-related affect as it has been presented in CERME conferences. The work in CERME has contributed to the development of the theoretical framework for affect. The terminology has both been refined and extended. For example, we can see the emergence of social theories for affect. The chapter will also report the results of a network analysis identifying the different schools of affect research in CERME and their most influential theorists. The chapter includes also a discussion on the different types of stabilities related to affect and an overview of trends in research methods applied. The chapter ends with a recommendation for important directions for future research on mathematics related affect.

Introduction

At this point, the relevance of affect in the mathematical teaching and learning process is recognized in the community of math educators. Looking back, and in particular looking at the progress of the research about affect through the analysis of CERME papers, appears to be particularly significant because the history and the development of the thematic working group on affect and mathematical thinking reflects in great part the evolution of the field of affect in mathematics education.

The initial caution of the community in considering affective issues is showed by the fact that CERME has hosted a thematic working group on affect only since its third edition. Particularly interesting that the first contributes on affective aspects appeared already in proceedings of CERME 1 (Zan & Poli, 1999), but submitted for the group called "mathematical thinking and learning as cognitive processes"! As well as, the continuous growing of attention to affective issues in the mathematics education community is revealed also by the increase of submissions for the relative thematic working group (28 in the last edition, CERME 9).

The ideas presented and discussed in the CERME conferences had a crucial role in the development of the field of affect. As we will try to show in the following paragraphs, significant results, theoretical and methodological issues, and new lines of research emerged by the discussion developed within the group Affect and mathematical thinking.

Development of theories in the field

The affect group in CERME has spent a lot of time and energy discussing the conceptual framework and terminology, leading to more extensive theorization of the area:

> "[The discussions] increased our awareness of being specific about the concepts that we use. We have realized that it is not sufficient to give definitions of the concepts that are being used in a particular study, but we have to explicate their relations to the other dimensions of affect research as well." (Hannula, 2011, p. 41).

There are three theoretical frameworks that have been influential in CERME for structuring the area of affect. The first is McLeod's (1992) framework that identified three main topics of research in mathematics related affect: emotions, attitudes, and beliefs (Figure 1). Moreover, the framework suggested that emotions are the most intensive, the least stable, and the least cognitive of the three, while beliefs are at the other end of the continuum and attitudes are in the middle.

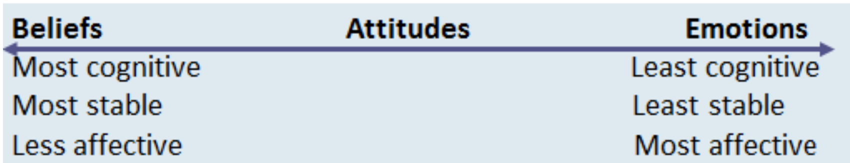

Figure 1: McLeod's (1992) framework on Affect

A significant step forward was the graphic representation of the conceptual field that Peter Op 't Eynde composed during CERME 5 (Figure 2*)*. This model captured new ideas discussed in CERME affect group: it recognized motivation as an important concept and identified different levels of social context.

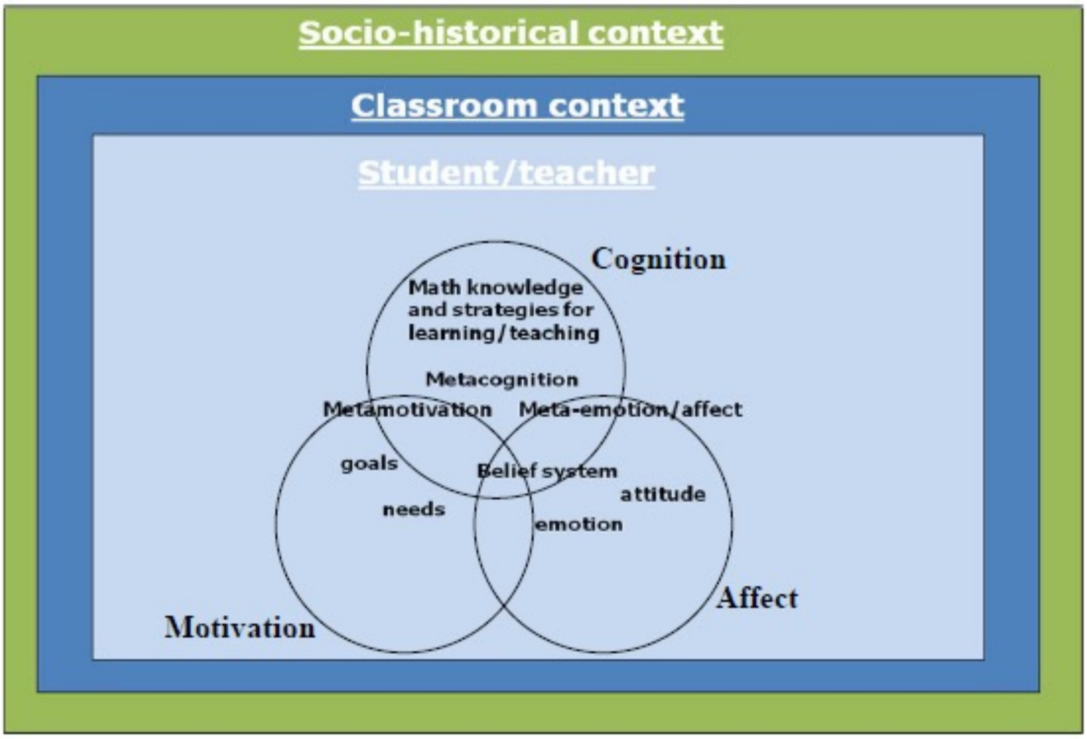

Figure 2. The different dimensions of mathematics-related affect and their relationships, presented at CERME 5 (Hannula, Op 't Eynde, Schlöglmann & Wedege, 2007, p. 204).

These ideas were further elaborated by Markku Hannula in his CERME plenary (2011) and in an article for CERME special issue of RME (2012). Hannula identified three dimensions that can be used to identify and define affective theoretical concepts (Figure 3). The first dimension recognizes that the concepts may be either cognitive (what one believes), affective (what one feels), or motivational (what one desires). The second, temporal dimension, separates state-type constructs that aim to describe dynamical processes, and trait-type constructs that aim to describe rather stable dispositions. The third dimension recognized the social turn (Lerman, 2000) in mathematics education research and also the embodied nature of affect, identifying three ontologically different traditions for affect-related research: psychological, social and embodied theories.

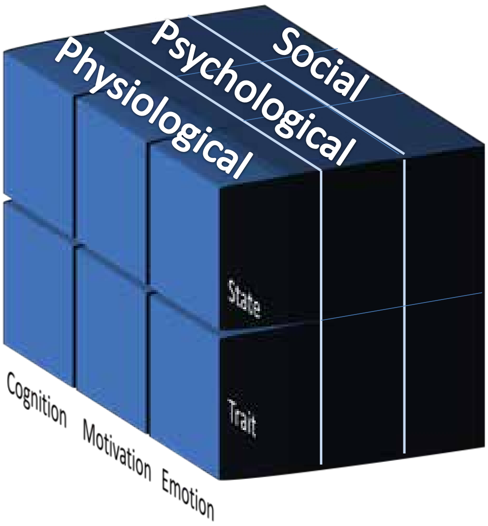

Figure 3. Hannula's (2011, p. 46; 2012) cube model of the three dimensions for affective constructs

Affect and Mathematical Thinking Group-Looking back

In order to empirically explore the relevance of different constructs and dimensions suggested above, we analyzed terminology for affect appearing in the titles of the 110 CERME affect papers (Table 1). We identified altogether 44 different affect terms, 26 of which appeared only once. The most frequently appearing terms were Belief, Affect(ive), Emotion, Attitude, Motivation, and Self-efficacy / Teaching efficacy. All these appeared already in CERME 3, except for Motivation. The frequencies of the term Attitude indicate a trend of decreasing popularity and the frequencies of the term Emotion indicate a sudden increased popularity in CERME 9.

Confe-rence	Papers	Affect terms	Unique terms	The most frequently appearing affect terms					
				Belief	Affect /ive	Emotion	Atti-tude	Motiva-tion	Self-efficacy / Teaching efficacy
CERME 3	10	5	0	2	4	2	3	0	2
CERME 4	11	8	2	0	1	0	4	0	0
CERME 5	19	15	5	6	0	1	2	4	1
CERME 6	14	14	1	4	1	1	1	3	1
CERME 7	11	14	6	2	1	0	1	0	1
CERME 8	17	13	6	5	0	1	1	3	2
CERME 9	28	15	6	3	8	7	0	2	2
Total	110	44	26	22	15	12	12	12	9

Table 1. Appearance of affect concepts in the titles of CERME affect papers.

New concepts have been also introduced to the group over the years. For example two interesting new affective constructs emerged in the discussion in CERME 6: the concept of personal meaning (Vollstedt, 2009), and the concept of teachers' emotional knowledge (Lavy & Shiriki, 2009).

Next, we present results of a network analysis of affect papers in CERME 4 to CERME 9 (more detailed description of the method and results can be found in Hannula & Garcia Moreno-Esteva, 2017). The analysis used graph theory to identify groups of papers citing the same authors. The results suggest ten groups: Foundation (30 papers), Self-Efficacy (11 papers), Motivation (11 papers), Teacher Development (8 papers), Academic Emotions (4 papers), Metacognition (4 papers), Teacher Beliefs (3 papers), Resilience (5 papers), Meaning (4 papers), and Identity (4 papers). As many of the groups are quite small a more detailed examination is useful.

The largest group, Foundation, was united by citing some key researchers in the field of mathematics related affect (e.g. McLeod, Schoenfeld, and Goldin) and some active CERME affect group participants (Hannula, Zan, Pehkonen, and Di Martino). Most of these papers used theoretical frameworks where affect (7 papers), attitude (8 papers), belief (7 papers), or emotion (10 papers) were among the key concepts. Rather than identifying a separate research tradition, this group represents the common ground largely shared by CERME affect papers. Looking at the other identified groups, we see that belief research appears in three groups: Foundation, Self-Efficacy, and Teacher Beliefs. Likewise, emotion appears in Foundation and Academic Emotions.

Taken together, the results confirm the results of the previous analysis of affect terms in article titles, identifying affect, belief, attitude, motivation, and emotion as the key theoretical concepts. Moreover, both analyses suggest that research on self-efficacy beliefs is somewhat separated from other belief research.

These results support the distinction between cognition (beliefs), motivation, and emotions as proposed by Op 't Eynde (Figure 2) and Hannula (Figure 3) models. Motivation was not included in the highly influential McLeod (1992) framework, and it emerged first in CERME 5. However, since then motivation has become an established key concept for studying mathematics affect. Among the several theoretical approaches that have been developed in the realm of educational psychology the most influential in CERME has been the achievement goal theory (Elliot, 1999). Pantziara and Philippou (2009; 2011) have investigated primary students' different motivational goals (mastery, performance and performance-avoidance) and found that mastery goals are related to positive affective variables (self-efficacy, interest) and behavior (achievement).

The separate group on Identity advocates the relevance for the distinction between psychological and sociological theories. The first papers that discussed identity (Kaasila, Hannula, Laine, & Pehkonen, 2005; Gómez-Chacón, 2005) were classified in the Foundation group, suggesting their need to relate to the dominating frameworks of that time, which has no longer been necessary in more recent CERME papers.

Moreover, the analysis suggests dynamics of change as a possible additional characterizing feature for research, exemplified by the identified groups Teacher Development and Resilience. This might add another category between the rapidly changing state aspect and relatively stable trait aspect in the temporal dimension of Hannula model (Figure 3).

Key findings of the CERME affect research

Looking closely at the results of the empirical studies discussed in Affect groups through all CERME meetings, one can recognize the main focus of research that have been discussed over the years. Research interest concerned (a) the structure of affect and the relation between the different affective variables (b) the relation between affect and achievement, (c) what makes students to continue studying mathematics beyond the compulsory levels, (d) the role of affect in mathematical problem solving and problem posing, (e) change in students affect, and (f) comparative studies. Some results in these areas are presented below.

(a) The structure of affect and the relation between the different affective variables

In this focus area, the results of the different studies concentrate on main affective concepts and the multidimensional structure of the affective domain. These variables included students' beliefs, self-efficacy beliefs, attitudes, emotions, fear, confidence, and motivation.

Goldin, Rösken and Törner (2010) state that beliefs have a structure, belong to a structured systems of beliefs, and are embedded in complex affective structures which are important to understanding students' and teachers' motivations and behavioral patterns. The results of many studies in the group with different aged group students supported this perspective and also revealed important patterns regarding the relation between positive beliefs and other affective variables but also the importance of social variables like the school context (e.g. teacher, grade) and gender, in the formation of different beliefs. In different CERME meetings researchers (Kıbrıslıoğlu & Haser, 2015; Rösken, Hannula, Pehkonen, Kaasila, & Laine, 2007; Kapetanas & Zahariades, 2007; Gagatsis, Panaoura, Deliyianni & Elia, 2009) developed beliefs scales in order to investigate students' beliefs in mathematics and mathematics learning/teaching. Some scales were more extended than others including various dimensions. In a closer look these scales included factors like beliefs about mathematics and mathematics learning, beliefs related to the personal aspect (self-efficacy, emotional expression, competence, effort related to mathematics) and also the social aspect, the role of the teacher and the family encouragement. The scales were used to investigate primary and secondary students' beliefs with some important findings.

The findings by Roösken et al. (2007) included significant differences in the beliefs subscales related to 11th grade students of general or advanced courses with students in advanced courses having more positive beliefs. The study by Hannula (2009) revealed that 11th graders of the same class tended to have similar effort, enjoyment of mathematics, and evaluation of teacher while their mathematical confidence was influenced by gender and their perception of their competence mainly related to their achievement in mathematics. The study by Kapetanas and Zahariades (2007) traced differences in10th-12th grade students' beliefs related to the type of school (public, private, and technical) and their mathematical ability. The study's results supported the pattern that positive affect (e.g. love of mathematics) is correlated positively with high performance in mathematics. In the same vein, Gagatsis et al. (2009) found that students of grades 5-8 with high mathematical performance had at the same time positive beliefs for the use of representations and high self-efficacy beliefs and Panaoura, Deliyianni, Gagatsis & Elia (2011) showed differences in students' beliefs in respect to school grade. Tuohilampi (2011) combined self-beliefs and motivational theories (achievement goals) to investigate the discrepancy between real and ideal self.

The structure of attitudes received great attention in the discussions of affective meetings. Di Martino (2009) investigated the structure of attitudes as emerged from the essays of 1600 students across grades 1-13. Three dimensions of the attitude construct emerged, the emotional disposition (concisely expressed by "I like/do not like maths"), an affective one (expressed by "to like" and "to adore") and one correlated with the idea of success in mathematics (expressed by "to understand" and "clever").

Research on the structure of mathematics-related emotions has been less active than respective research on attitudes and beliefs. Studies about emotions were interested in investigating different emotions at the same time, in investigating the intensity of emotions, factors that develop certain emotions and also emotional regulation strategies. Martinez-Sierra (2015) applied Ortony, Clore, and Collins' (1988) theory of the cognitive structure of emotions, which specifies eliciting conditions for each type of emotion and the variables that affect intensity. He found that the emotional experience of 54 high school students was characterized by satisfaction, disappointment, hope, fear, joy, distress, boredom, interest, pride, reproach, self-reproach, like, and dislike with different eliciting conditions. The results showed that all students' emotional experiences were based on their appraisal in terms of a goal structure presented in the mathematics classroom and in the school setting. Op 't Eynde De Corte, and Mercken (2007) concluded that students know and make use of six different categories of emotional regulation strategies in stressful school situations related to mathematics learning, including active and problem focused strategies as well as more emotion focused strategies. They found clear differences in the kind of strategies used by students depending on the situation confronted with, their familiarity with the stressful nature of this situation, the track level they are in, their age and gender.

(b) The relation between affect and achievement

The positive correlation between affective variables and mathematics achievement has been extensively discussed (Liljedahl & Hannula, 2016). In CERME affect meetings studies have verified this positive correlation between different affective variables. The studies reported positive correlation between primary students' attitudes and mathematics achievement (Nicolaidou & Philippou, 2005) and self-efficacy beliefs and mathematics achievement (Nicolaidou & Philippou, 2005; Sofokleous & Gagatsis, 2009) in different areas of mathematics (problem solving, geometry). In both studies the affective variables were the predictors of mathematics achievement. In the same vein, Pantziara & Philippou (2007) showed that 6^{th} grade students in the upper levels of conceptual understanding of fractions were characterised by less fear of failure and more mastery goals and self-efficacy than students in the lower levels of conceptual understanding. Schukajlow (2015) investigated the relation between 9^{th} and 10^{th} graders' performance and boredom using task-unspecific and task-specific scales. While the results were not univocal, some findings revealed that students who achieved higher scores reported lower boredom across different types of problems.

(c) What makes students to continue studying mathematics

Dropping out of mathematics and especially advanced mathematics has become a major concern for society (Moscucci, Piccione, Rinaldi & Simoni, 2005). Moreover, several reports indicate a low rate of tertiary students around the world that are enrolled in science, technology, engineering and mathematics (STEM) related careers and even lower rate refers to women (Sánchez Aguilar, Romo Vázquez, Rosas Mendoza, Molina Zavaleta, & Castañeda 2013). Moscucci et al. (2005) underpinning the role of affect in this situation, showed that when a student failed at school (i.e. is a dropout) then, in the same school year, s/he failed in mathematics. Assuming that mathematical failure always follows mathematical discomfort they ended that discomfort elimination will lead to the reduction of more than 50% the critical number of the drop-out variables. Other studies have investigated factors that influence students' enrolment in advanced mathematical courses. Factors referred to students own awareness about their competence in mathematics, their future expectations but also to their social context-teachers and relatives (Kleanthous & Williams; 2011; Sánchez Aguilar et al., 2013).

(d) The role of affect in mathematical problem solving and problem posing

One important area of research on mathematics related affect is the role of different affective constructs in problem solving. Some quantitative research reports in the group investigated the relationship between multiple affective variables and problem solving and posing. Nicolaou and Philippou (2007) found in their study with students of grade 5 and 6 that students' perceived efficacy to construct problems was a stronger predictor of their ability in problem posing and of their general mathematics achievement than their attitudes.

Research describing students' emotions and their changes during problem-solving, investigated factors behind the change, and the potential impact of this change on mathematical activity that may provide significant indications between affective and cognitive factors. Antognazza, Di Martino, Pellandini and Sbaragli (2015) implemented a three phase approach to describe 91 primary students' emotions when dealing with problem solving. They revealed a distinction between positive or negative emotions perceived in a specific mathematics problem solving activity deriving from an assessment of the difficulty of the activity proposed (intrinsic aspects), or from more general aspects. Assuming the existence of a mutual influence of affective and cognitive factors, Furinghetti and Morselli (2005) revealed in their case study of a good problem solver who faced the proof of a statement in number theory, that among the elements that shaped the behavior of a good problem solver were aesthetic values and feelings of freedom in facing the problem. In the same vein, Viitala (2015) described a grade 9 high achiever's mathematical thinking through problem solving and mathematics related affect. The results revealed a successful, though quite unsure, problem solver whose affective state (connected to problem solving) seemed to tell the same story as her affective trait (view of mathematics). The differences between results on affective state and trait seemed to be connected mostly to emotions.

(e) Change in students' affect

Affective traits develop over time. CERME papers have discussed the dynamic (process) aspects for the theory for affect. Some CERME papers (e.g. Liljedahl, 2009; Stylianides & Stylianides, 2011) have discussed the nature of stability and malleability of affect. In CERME 6, the affect group identified and reported four different aspects of stability (Hannula, Pantziara, Wæge, & Schlöglmann, 2009): 1) The *state* and *trait* aspects of affect; 2) Resistance to change; 3) Robustness of constructs; and 4) Relative stability in relation to other persons. More specifically, the relationship between affect and achievement is reciprocal, i.e. poor performance will influence affect to become more negative and more negative affect predicts poorer future achievement.

In the CERME affect meetings this development was discussed both in the context of students and teachers. Remaining in students' perspective in this area of research, one identifies the changes in students' affect as they move to upper school grades, or the importance of the classroom microculture on students' affect. Change in students affect was also observed through their engagement in structured interventions like problem solving and modelling cases or through specific instructional practices. It is also known that social factors, such as gender and SES influence affect.

Past research evidently indicates that students' mathematics-related affect develops detrimentally during school years. A decline in students' positive affect was documented by the studies of Athanasiou and Philippou (2009) during the transition to secondary school. Elementary school students endorsed more praise and token goals and social motivational orientations whereas middle school students endorsed competition goals and performance motivational orientations. Tuohilampi, Näveri and Laine (2015) trying to prevent this decline in students' affect applied a three-year intervention designed to improve primary school students' problem solving skills, and their mathematics-related affect. The impact was restricted but crucial: girls' affect regarding mathematics decreased less in the intervention group.

Several studies reported positive change in students' affect after structured interventions through problem solving (Marcou & Lerman, 2007) and modelling (Schukajlow & Krug 2013) in primary and secondary students. Barnes (20015) reported on a small-scale intervention that explored perseverance in mathematical reasoning in children aged 10–11. The intervention facilitated children's provisional use of representations during mathematical reasoning activities. The findings suggest improved perseverance because of the effect the intervention seemed to have on the bidirectional interplay between affect and cognition.

Students' affect are influenced by the learning context and the teacher (Liljedahl & Hannula, 2016). However, the experiences of students in one class may differ and the development of their affects may follow very different paths. Some studies (e.g.Vankúš, 2007) showed increase in students affect after implementing some new practices like didactical games and humor in the mathematics classroom.

Helmane (2015) selected the factors facilitating positive and negative emotions while teaching mathematics in primary school. In most cases, the students' interest and joy were aroused by the opportunity to use visual aids, play didactic games and the teacher's positive attitude in mathematics lessons. In several cases, the students experienced negative emotions such as fear, shame and sorrow in mathematics lessons. These negative emotions caused by the situations related to a student's incompetence, failure in doing a certain mathematics task as well as by the cases when students encountered with a negative assessment of their work and the comparison of their work with that of the other students.

Studies in the realms of motivation showed that students' motivation for learning mathematics, although it is considered relatively stable, can be influenced and altered by changes in the teaching approach (Pantziara & Philippou, 2009; Wæge, 2009).

(i) Comparative studies

Comparative studies were rear in the group meetings. Andrews, Mantecón, Op 't Eynde, and Sayers (2007) discussed the effectiveness of a revised instrument as a means of discriminating between the mathematics-related beliefs of students from schools in England and Spain, and examined its potential for distinguishing between gender and age. The results suggested that the scale served all the purposes well, highlighting a number of culturally-, age- and gender-related differences. Pepin's paper (2011) reported on a comparative study of English and Norwegian secondary students' attitudes toward mathematics. Analyzing students' comments (on the questionnaires) the author contended that student attitude is embedded and shaped by the context (classroom and larger school environment) in which it develops.

The development of methodology in the field of affect

The methodological issues have always had large space in the discussions developed in the Affect thematic working group in CERME. Methodological issues are important for all types of research, but particularly critical in the field of affect (Evans, 2003). Many reflections emerged in these discussions affected the general development of the field in mathematics education.

First, the literature showed the lack of a generally accepted conceptualization of what really the principal constructs (attitudes, beliefs, emotions, motivation) means: the different constructs tended to be defined implicitly and a posteriori through the instruments used to measure it (Furinghetti & Pehkonen, 2002; Di Martino & Zan, 2001) and, a sharp delimitation between these constructs did not exist. Schloeglmann (2003) underlined that the research methods traditionally used in the field of affect could not establish a distinction between the above categories and he used this argument to encourage the development of new research methods and approaches. In particular, he suggested considering methods developed in different domains, such as neuroscience.

Second, the nature of affective constructs makes it difficult to infer them. There are essentially two schools of thought about that: one sees affective constructs as an inner awareness or process of interpretation of events rather than an overt behavior and consequently they are not directly observable and, moreover, individuals themselves are often not conscious of these processes (Panaoura & Philippou, 2003). Another school of thought sees affective constructs (such as attitudes and beliefs) not as a quality of an individual but "rather as a construct of an observer's desire to formulate a story to account for observation" (Ruffel, Allen & Mason, 1998, p. 1). In both views, the problems connected with the methodology are evident. In particular, it seems evident that the researchers' choices about methodology (and also the context of the study) can condition and constrain the findings (Pantziara, Wæge, Di Martino & Rösken-Winter, 2013).

Third, the presence of a dynamic issue in the research on affect: the distinction between rapidly changing affective states and relatively stable affective traits. Schloeglmann (2003) argued that quantitative methods reveal stable and less intense categories, while qualitative methods are able to grasp quickly changing and very intense reactions. Actually, the first qualitative studies presented in our CERME group were typically case studies (Pietilä, 2003; Furinghetti & Morselli, 2005) or small sample studies (Liljedahl, Rolka & Rösken, 2007) developed to observe *changes in action*. But, a more deepened analysis (Hannula, 2011) shows that the majority of the affective constructs have both a state aspect and a trait aspect. This is probably one of the main reasons for the emergence of studies that use mixed methods.

Fourth, new issues and new goals in the research on affect have been identified in the last 20 years. Also in this case, we can highlight two main directions: the first one follows the traditional approach in the field of affect, searching for causal relationship between affective variables and mathematical performances or behavior. In this frame, the crucial action is to measure, privileging quantitative methods. It demands isolating, clearly identifying, and measuring variables in order to interpret statistical results. A necessary part of the studies conducted within this frame is developing the means for the efficient measurement of affective constructs, but also of mathematical performance. The second one – following the gradual affirmation of the interpretive paradigm in the social sciences – abandons the goals of explaining behaviour through measurements and of determining general rules based on a cause-effect model to describe the interaction between affective and cognitive constructs in mathematics education, and focuses on trying to make sense of the observed phenomena from the perspective of participants. This implies a significant shift in focus and, in particular, a movement towards the use of qualitative approaches (Evans, Hannula, Philippou & Zan, 2003).

An evident consequence of these considerations is the trend (from CERME 3 to more recent editions of CERME) towards the use of mixed methods (quantitative and qualitative) in the research on affect, overcoming the initial preponderance of quantitative methods. Moreover, the shift of the focus from the description of a phenomenon to the interpretation of the same phenomenon intensifies the attention on how the collected data is interpreted (Di Martino, Gómez-Chacón, Liljedahl, Morselli, Pantziara & Schukajlow, 2015).

Summarizing, the ideas discussed in the congresses of ERME have highlighted the main critical aspects about methodological issues in the field of affect, contributing to the refinement of methods and, more in general, to the development of the field.

Concerning quantitative methods, the development of new tools is particularly critical because validity is essential. So the majority of the quantitative papers presented in past CERME used classic and consolidated questionnaires and scales, but two interesting trends emerge: the first one, coherent with the consideration of the complexity of the affective factors, is the trend to modify and combine two or more scales for the same studies (Pantziara and Philippou, 2011); the second is the trend to adopt the more complex computational tools that have become available to analyze the data (Mosvold, Fauskanger, Bjuland & Jakobsen, 2011).

Whereas, for what concerns qualitative approaches, CERME papers often introduced new methods to collect data. An exemplary case is offered by the Kaasila, Hannula, Laine and Pehkonen's paper (2005) in CERME 4: it discusses the potential of autobiographical narratives in order to reconstruct students or teachers' mathematical identity. According to Connelly and Clandinin (1990), the key assumption is that humans are storytelling organisms who, individually and socially, lead storied lives. The study of narrative, therefore, is the study of the ways humans experience the world.

Affect and Mathematical Thinking Group-Looking ahead

The research on mathematics-related affect has repeatedly raised the terminological issues as a problem (e.g. Furinghetti & Pehkonen, 2002; Hannula, 2011; 2012). Sometimes researchers use the same term with different meanings, while at other times different terms seem to indicate the same construct. For example, Di Martino and Zan (2001) have discussed thoroughly the use of "attitude" both as a broad umbrella term, and as a more specific concept. The problem persists as the field still has difficulties to solidify a shared terminology. This problem is clearly related to the cumulative nature of the research in mathematics education, and therefore to the need that new research builds on a critical analysis of the previous research. For its organization, CERME group has been and will be an important place to highlight this fundamental issue and to tackle it. Yet, it is important to keep a way open for new concepts to emerge. It seems clear, at least in retrospect, that motivation and identity were terms that were necessary for the research field. It seems reasonable that we need specific terms, for example, for "Aesthetic" (Müller-Hill & Spies, 2015), "Perseverance" (Barnes, 2015), and "Resilience" (Lee & Johnston-Wilder, 2011).

While there has been quite substantial integrative work aiming to identify the structure of affective traits, there has been less efforts to find a unifying theory of affective traits and dynamic states. We see this integration between research on states and traits as an important goal for our research area.

Despite the improved understanding of mathematics related affect, the general trend still is that enjoyment of mathematics decreases over the school years. We need to develop teaching approaches that promote a positive relationship with mathematics without compromising understanding of concepts. Such approaches should be tested through systematic longitudinal intervention studies, and across different cultural and social contexts

There are three specific methodological possibilities that can open yet new understandings of the dynamics of mathematics-related affect. The first possibility would be to analyze the dynamics of group level processes: How does the teacher initiate and maintain excitement and good working climate in the class? What kind of processes lead to the 'energy' of the class being lost? Concepts like classroom climate would be useful concepts for this kind of analysis. The other new methodological possibility is to implement physiological measures (e.g. heart rate monitoring) to gain a continuous indication of participants' affective states. These methods have been used a long time in laboratories, but have only recently become more affordable and usable in actual classrooms.

The third methodological possibility is highly associated with CERME spirit of collaboration. Comparative research on mathematics-related affect has confirmed that while some research findings about affect are universal, some other findings are contextual. Therefore, there is need to examine which results about affect are transferable to different sociocultural contexts, and Europe with its diverse educational systems and linguistic groups is a wonderful testbed for such comparative studies.

REFERENCES

Andrews, P., Mantecón, J.D., Op ‘t Eynde, P., & Sayers, J. (2007). Evaluating the sensitivity of the refined mathematics-related beliefs questionnaire to nationality, gender and age. *CERME 5* (pp. 209-218).

Antognazza, D., Di Martino, P., Pellandini, A., & Sbaragli, S. (2015). The flow of emotions in primary school problem solving. *CERME 9* (pp. 1116-1122).

Athanasiou, C., & Philippou, G. (2009). The effects of changes in the perceived classroom social culture on motivation in mathematics across transitions. *CERME 6* (pp. 114-123).

Barnes, A. (2015). Improving children’s perseverance in mathematical reasoning: Creating conditions for productive interplay between cognition and affect. *CERME 9* (pp. 1131-1138).

Connelly, F. M., & Clandinin, D.J. (1990). Stories of experience and narrative inquiry. Educational Researcher, 19 (5), 2-14.Furinghetti, F. & Morselli, F. (2005). Reflections on creativity: the case of a good problem solver. *CERME 4* (pp. 184-193).

Goldin, G., Rösken, B., & Törner, G. (2010). Professionalism and identity - on the structured nature of affect, motivation, and beliefs. In M. M. F. Pinto & T. F. Kawasaki (Eds.) *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education, PME 34, Belo Horizonte, Brazil, July, 2010.* Volume 1, pp. 250-256. Belo Horizonte: PME.

Di Martino, P. (2009). “Maths and me”: software analysis of narrative data about attitude towards math. *CERME 6* (pp. 54-63).

Di Martino, P., & Zan, R. (2001). Attitude toward mathematics: Some theoretical issues. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th conference of the IGPME* (Vol. 3, pp. 351–358). Utrecht, The Netherlands .

Di Martino, P, Gómez-Chacón, I., Liljedahl, P., Morselli F., Pantziara, M. and Schukajlow, S. (2015). Introduction to the papers of TWG08: Affect and mathematical thinking. *CERME 9* (pp. 1104-1107).

Elliot, A. (1999). Approach and avoidance motivation and achievement goals. *Educational Psychologist, 34*, 169-189.

Evans, J. (2003). Methods and findings in research on affect and emotion in mathematics education. *CERME 3*.

Evans, J., Hannula, M., Philippou, G. & Zan, R. (2003). Introduction to the papers of the thematic working group Affect and Mathematical Thinking. *CERME 3*.

Furinghetti, F. & Morselli, F. (2005). Reflections on creativity: the case of a good problem solver. *CERME 4* (pp. 184-193).

Furinghetti, F., & Pehkonen, E. (2002). Rethinking characterizations of beliefs. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 39–57). Dordrecht: Kluwer.

Gagatsis, A., Panaoura, A., Deliyianni, E. & Elia, I. (2009). Students' beliefs about the use of representations in the learning of fractions. *CERME 6* (pp. 64-73).

Gómez-Chacón, I. M. (2005). Affect, mathematical thinking and intercultural learning. A study on educational practice. *CERME 5* (pp. 194-204).

Hannula, M.S. (2009). The effect of achievement, gender and classroom context on upper secondary students' mathematical beliefs. *CERME 6* (pp. 34-41).

Hannula, M.S. (2011). The structure and dynamics of affect in mathematical thinking and learning. *CERME 7* (pp. 34-60).

Hannula, M.S. (2012). Exploring new dimensions of mathematics related affect: Embodied and social theories. *Research in Mathematics Education, 14*(2), 137-161.

Hannula, M.S., & Garcia Moreno-Esteva (2017). Identifying subgroups of CERME affect research papers. *CERME 10*.

Hannula, M.S., Op 't Eynde, P., Schlöglmann, W., & Wedege, T. (2007). Affect and mathematical thinking. *CERME 5* (pp. 202 – 208).

Hannula, M.S., Pantziara, M., Wæge, K., & Schlôglmann, W. (2009). Introduction: Multimethod approaches to the multidimensional affect in mathematics education. *CERME 6* (pp. 28 – 33).

Helmane, I. (2015). Basic emotions of primary school pupils in mathematics lessons. *CERME* 9 (pp. 1195-1201).

Kaasila, R., Hannula, M.S., Laine, A. & Pehkonen, E. (2005). Autobiographical narratives, identity and view of mathematics. *CERME 4* (pp. 215-224).

Kapetanas, E., & Zachariades, T. (2007). Students' beliefs and attitudes concerning mathematics and their effect on mathematical ability. *CERME 5* (pp. 258-267).

Kıbrıslıoğlu, N., & Haser, Ç. (2015). Development of mathematics-related beliefsscale for the 5th grade students in Turkey. *CERME 9* (pp. 1202-1208).

Kleanthous, E., & Williams, J. (2011). Students' dispositions to study further mathematics in higher education: the effect of students' mathematics self-efficacy. *CERME 7* (pp. 1229-1238).

Lavy, I. & Shriki, A. (2009). Emotional knowledge of mathematics teachers – retrospective perspectives of two case studies. *CERME 6* (pp. 134-143).

Lee, C., & Johnston-Wilder, S. (2011). The pupils' voice in creating a mathematically resilient community of learners. *CERME 7* (pp. 1189-1198).

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Westport, CN: Ablex.

Liljedahl, P. (2009). Changing beliefs as changing perspective. *CERME 6* pp. 44-53.

Liljedahl, P., Rolka, K. & Rösken, B. (2007). Belief change as conceptual change. *CERME 5* (pp. 278-287).

Liljedahl, P., & Hannula, M.S. (2016). Research on mathematics-related affect in PME 2005-2015. In A. Gutierrez, G. C. Leder & P. Boero (Eds.) *The Second Handbook of Research on the Psychology of Mathematics Education* (pp. 417-446). Netherlands: Sense Publisher.

Marcou, A., & Lerman, S. (2007). Changes in students' motivational beliefs and performance in a self-regulated mathematical problem-solving environment. *CERME 5* (pp. 288-297).

Martínez-Sierra, G. (2015). Students' emotional experiences in high school mathematics classroom. *CERME 9* (pp. 1181-1187).

McLeod, D. B. (1992). Research on affect in mathematics education: a reconceptualization. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Learning and Teaching* (pp. 575-596). New York: MacMillan.

Moscucci, M., Piccione, M., Rinaldi, M.G., & Simoni, S. (2005). Mathematical discomfort and school drop-out in Italy. *CERME 4* (pp. 245-254).

Mosvold, R., Fauskanger, J., Bjuland, R. & Jakobsen, A. (2011). Using content analysis to investigate student teachers' beliefs about pupils. *CERME 8* (pp. 1389-1398).

Müller-Hill, E., & Spies, S. (2015). On the role of affect for sense making in learning mathematics - aesthetic experiences in problem solving processes*.* *CERME 9* (pp. 1245-1251).

Nicolaidou, M., & Philippou, G. (2005). Attitudes towards mathematics, self-efficacy and achievement in problem-solving*.* *CERME 3*

Op 't Eynde, P., De Corte, E., Mercken, I. (2007). Students' self regulation of emotions in mathematics learning. *CERME 5* (pp.318-3280).

Ortony, A., Clore, G. L., & Collins, A. (1988). *The cognitive structure of emotions*. Cambridge, UK: Cambridge University Press.

Panaoura, A. & Philippou, G. (2003). The measurement of young pupils' metacognitive ability in mathematics: the case of self-representation and self-evaluation. *CERME 3*.

Panaoura A., Deliyianni, E., Gagatsis, A., & Elia, I. (2011). Self – about using representations while solving geometrical problems. *CERME 7*, (pp. 1167-1178).

Pantziara, M., Pitta-Pantazi D., & Philippou, G. (2007) Is motivation analogous to cognition? *CERME 5* (pp. 339-348).

Pantziara, M., & Philippou, G. (2009). Endorsing motivation. Identification of Instructional practices. *CERME 6* (pp. 106-113).

Pantziara, M. & Philippou, G. (2011). Fear of failure in mathematics. What are the sources? *CERME 7* (pp. 1269-1278).

Pantziara, M., Wæge, K., Di Martino, P. & Rösken-Winter B. (2013). Introduction to the papers of the thematic working group Affect and Mathematical Thinking. *CERME 8* (pp. 1272-1278).

Pietilä, A. (2003). Fulfilling the criteria for a good mathematics teacher – the case of one student. *CERME 3*.

Rösken, B., Hannula, M.S., Pehkonen, E., Kaasila,R.,& Laine, A. (2007). Identifying dimensions of students' view of mathematics. Proceedings of *CERME 5* (pp. 349-358).

Ruffel, M., Mason, J. & Allen, B. (1998). Studying attitude to mathematics. *Educational Studies in Mathematics, 35*, 1–18.

Sánchez Aguilar, M., Romo Vázquez A, Rosas Mendoza A, Molina Zavaleta J. G., & Castañeda Alonso A. (2013). Factors motivating the choice of mathematics as a career among Mexican female students. *CERME 8* (pp. 1409-1418).

Schloeglmann, W. (2003). Can neuroscience help us better understand affective reactions in mathematics learning? *CERME 3*.

Schukajlow, S. (2015). Is boredom important for students' performance? *CERME 9* (pp. 1273-1279).

Schukajlow, S., & Krug, A. (2013). Uncertainty orientation, preference for solving task with multiple solutions and modelling. *CERME 8* (pp. 1428-1437).

Stylianides, G., & Stylianides, A. (2011). An intervention on students' problem-solving beliefs. *CERME 7* (pp. 1209-1218)

Tuohilampi, L. (2011). An examination of the connections between self discrepancies and effort, enjoyment and grades in mathematics. *CERME 7* (pp. 1239-1248).

Tuohilampi, L., Näveri, L., & Laine, A. (2015). The restricted yet crucial impact of an intervention on pupils' mathematics-related affect. CERME 9 (pp. 1287-1293).

Vankúš, P. (2007). Influence of didactical games on pupils' attitudes towards mathematics and process of its teaching. *CERME 5* (pp. 369-378).

Viitala, H. (2015). Emma's mathematical thinking, problem solving and affect. *CERME 9* (pp. 1294-1300)

Vollstedt, M. (2009). "After I do more exercise, I won't feel scared anymore" – examples of personal meaning from Hong Kong *CERME 6* (pp. 124-135).

Wæge, K. (2009). Students' motivation for learning mathematics in terms of needs and goals. *CEMRE 6* (pp. 84-93).

Zan, R. & Poli, P. (1999) Winning beliefs in mathematical problem solving. *CERME 1* (pp. 97-104).