CHAPTER 7: APPLICATIONS AND MODELLING FOR THE ERME BOOK

# ANALYZING RELATIONS BETWEEN DEVELOPMENT OF TEACHING PRACTICES AND RESEARCH IN THE FIELD OF TEACHING AND LEARING APPLICATIONS AND MODELLING

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## Modelling and application as a field of research

The field of research and development related to the teaching and learning of applications and mathematical modelling is well-established and in vivid development with the *International Community of Teachers of Mathematical Modelling and Application* (ICTMA) as its backbone. ICTMA is an affiliated study group of the *International Commission on Mathematical Instruction* (ICMI) with a topic study group and a general meeting at the ICME congresses. In addition ICTMA organises biannual international conferences resulting in published study volumes in the Springer book series *International perspectives on the teaching and learning of mathematical modelling*; see http://www.ictma15.edu.au/.

Since CERME 4 in 2005 in Sant Felie de Guíxols, Spain, the conferences of ERME have included a thematic working group on applications and modelling (WGAM), and the biannual CERMEs have from there on been part of the infrastructure of the ICTMA community. It was the then president of ICMTA Gabriele Kaiser, Germany, who took the initiative for the formation of the WGAM. The organising teams have typically been recruited among the European members of the ICTMA community, and the WGAMs have attracted experienced researchers as well as welcoming new researcher to the field, primarily of course from Europe.

In general, the field of research related to the teaching and learning of modelling and application has developed in close interplay with the development of the practices of teaching mathematics including these elements. During the latest decades research and developmental work in many European countries have influenced the inclusion of applications of models and modelling in their mathematics curricula, particularly at the secondary level. Research has clarified arguments for including models and modelling in general mathematics education; conceptualized modelling competence at different educational levels; identified teaching and learning obstacles related to models and modelling; and have exemplified the potential for enhancing the students' conceptual learning through modelling activities (Niss et al., 2007). Moreover, various theoretical based methodologies for developing and implementing practices of teaching modelling and applications in collaboration between researchers, educators and teachers have been developed and tested. The research field has to large extend developed coherent theoretical frameworks for justifying, designing and implementing modelling and applications in mathematics teaching at different levels in the educational system.

In his plenary addressed at ICME-12 Werner Blum surveyed the achievements in the field from the perspective of what it tells us about quality of the teaching of applications and modelling at secondary level. Based on empirical findings, Blum identified ten important aspects of a successful and productive teaching methodology (Blum, 2015, 83-86). However, he rounds off the list with the following remark:

> … I would like to emphasise that all these efforts will not be sufficient to assign applications and modelling its proper place in curricula and classrooms and to ensure effective and sustainable learning. The implementation of applications and modelling has to take place systemically, with all system components collaborating closely: curricula, standards, instruction, assessment and evaluation, and teacher education. (Blum, 2015, p. 87)

Thus, despite the progress made in research and its influence on curricula, there are still major challenges concerning the development of practices of teaching modelling and applications as an integrated element in mathematics teaching in general and higher tertiary education. A necessary, but of course not sufficient, condition for overcoming these challenges is to further develop the interplay between research and the development of teaching practices. This challenge is clearly reflected in the contributions and discussion at the WGAMs. Therefore, we have decided to focus our analyses of the WGAM contributions on exactly the interplay between research and the development of practices of teaching mathematical modelling and applications.

## Characterising the contributions from the WGAMs

In the six congresses CERME 4-9 there have been in total 102 papers (not including 5 shorter introductory papers) and 6 posters presented on applications and mathematical modelling. Disregarding the temporal break in the trend at CERME 7, the WGAMs have had an increasing number of contributions; see figure 1.

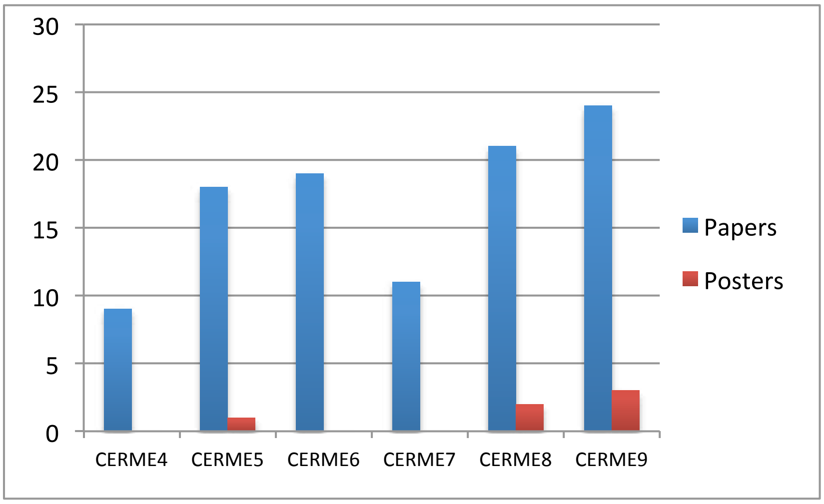


Figure 1: The number of contribution to WGAM at CERME 4-9.

The contributors are from 23 different countries from four continents, although 81% of the contributors are affiliated with European countries. Overall 24% of the contributors are affiliated with Germany, 17% with Spain, and around 6% with Denmark, Sweden, The Netherlands and United Kingdom respectively. The most non-European contributing country has been US (7%). On average each contribution have two authors, however, 35% of the contributions are single-authored. It is interesting to note that the proportion of single-authored contributions has decreased (except for CERME 7) over the years and at CERME 9 more that 80% of the contributions had two or more authors.

The WGAM contributions witness a broad shared understanding of the basic notions and the research achievements, although the field is still being researched within different theoretical frameworks or perspectives. Some of these differences seem to be founded in national traditions and preferences. The most dominant ones being the German and Scandinavian holistic and project oriented approach for developing and researching modelling competence (Blomhøj & Jensen, 2003; Frejd, 2011), the French and Spanish ATD based approach for theorizing the teaching and learning of mathematics in general (García et al., 2006; Barquero, Serrano, & Serrano, 2015), the Realistic Mathematics Education perspective on modelling applied by many Dutch researchers and others (Treffers, 1987; Vos, 2005). However, there are also other theoretical perspectives which can be identified across the national trends. The cognitive perspective on modelling (Borromeo Ferri, 2006; Roorda, Vos, & Goedhart, 2007) and the critical mathematics education approach towards the teaching and learning of applications and modelling (Skovsmose, 1994; Barbosa, 2005) are two examples hereof. In addition, recently there have also been an increasing number of WGAM contributions drawing on and using the so-called models and modelling perspective on teaching and learning of mathematics based on the modelling eliciting activities developed be Lesh and colleagues (Lesh & Doerr, 2003; Sriraman, 2005). It is possible to find examples of the use of all these different frameworks for researching the teaching and learning of modelling and applications among the WGAM contributions. In fact the second references given for each of the frameworks mentioned above are to a WGAM paper.

Already at CERME 4 and 5 the diversity of theoretical perspective and approaches used in the contributions to the WGAM as well as in the research field more generally were identified and discussed. This work resulted in a schematic categorisation of research perspectives further developed and presented in the introduction to the WGAM proceedings from CERME 5 (Kaiser et al., 2007). This work initiated at CERME 4 was also used to structure and discuss the status and developments in field in two thematic issues of ZDM in 2006 (no. 38 and 39), see Kaiser et al. (2006). This type of general characterisations of the research can facilitate the formation of a shared understanding of the field and move the discussions in directions of challenges to be addressed and away from a pro and contra discourse concerning particular theoretical perspectives and frameworks. In our opinion the work and discussions at the WGAMs have had exactly that effect within the research field.

However, although representing a large diversity in theoretical perspectives, the WGAM contributions have many commonalities. First of all, to a high extend there is a shared understanding in the papers of basic notions such as mathematical model, modelling process and modelling competency, and of the principle difficulties related to teaching and learning of modelling and applications in mathematics teaching in general education.

Moreover, many papers are situated within the development and testing of didactical designs for teaching modelling and applications in specific educational contexts. As elaborated on below over 75% of the circa hundred papers analysed have an empirical element connected to the development of teaching practice primarily at secondary level or to (the first year of) tertiary teaching in various mathematical based educations. In addition, most of the theoretical papers have explicit connections to didactical designs and/or to specific practices of teaching, although in this context typically used for illustration purposes or as basis for theorising.

To some degree the focus on the practices of teaching modelling and applications found in WGAM reflects the general situation in the research field. However, the very format for CERME contributions also encourages research papers reporting on small scale developmental projects or parts of larger projects in progress. Some of the papers – especially among those related to tertiary education – are reporting developmental projects related to (one of) the author(s) own teaching environment. Also, quite a few of the papers – say around 20% – are related to ongoing or recently finished Ph.D.-projects. The close connection to teaching practice is in general framing the WGAMs work and discussions. In our view, these features and a general inclusive and supporting atmosphere makes the WGAM accessible for newcomers, and serves as a possible entrance point for new researchers to the international research community related to the teaching and learning of modelling and applications.

## Structuring the analyses of the WGAM contributions

Returning to the main challenge pinpointed by Blum (2015), namely, for research to support a more systemic integration of applications and modelling throughout the educational system, we have decided to survey the WGAM papers according to the relation between the development of teaching practice and the use and development of theory.

In many European countries secondary education in general serves both the purpose of general education for citizenship in a democracy and the preparation for further education. The same fundamental duality is found in the educational aims for including applications and modelling in mathematics teaching in general education: the duality between on the one hand the aim of developing students' competences to setup, applied, analyze and critique models and their applications in contexts, and on the other hand the aim of motivating and supporting the students' learning of mathematics through modelling activities. Especially at the secondary level, this duality is important because both aims are often being pursed at the same time in classrooms.

In our view the relation between theory and teaching practice stands differently with regard to these two educational aims. Although often not stated explicitly in the WGAM contributions, we found in surveying the papers that most of them have a clear focus on one of these aims. In addition, in nearly all the papers there is a dominant perspective on either the development of teaching practice in a particular educational context or on the use or development of theory capturing potentials and difficulties concerning the teaching and learning of applications and modelling. Therefore, in our analysis of the WGAM contribution we distinguish between these two educational aims related to the applications and modelling in general mathematics teaching, and cross this dimension with the division between a focus either on the development of teaching practices, or a focus on the use or development of theories. This results in the 2x2 matrix seen in figure 2, which we used to structure our analysis.

| Modelling as / Focusing on development of | a means for the learning of mathematics | an educational aim in its own right |
|---|---|---|
| the practices of teaching mathematical modelling | (1) Integrating modelling in the teaching of mathematics | (2) Developing, implementing and analysing modelling projects and activities in teaching practice |
| theories on the teaching and learning of modelling | (3) Using and developing theories on the learning of mathematics connected to modelling | (4) Developing theories on the teaching and learning of modelling competency |

Figure 2: A matrix outlining the four categories of research on the teaching and learning of applications and modelling with respect to empirical or theoretical focus and the educational aims for teaching modelling.

In the following this matrix is used to categorise the WGAM contributions with the aim of characterising the interplay between research and development of teaching practices in the research conducted, and to identify potentials for further developments.

## Method

All the WGAM contributions were read and placed into one of the four categories in the 2x2 matrix introduced and discussed above (see figure 2). The reading of the papers forced and helped us to clarify and better articulate the difference between and within the different aspects of the two dimensions in the matrix. For the reader to gain some further insight in our conceptualization of the four categories and how we have characterised the WGAM papers, we will now briefly explain and elaborate on these differences.

The divide between the two rows in the matrix is related to the existence and role of empirical data within the contributions. Most of the papers categorised as belonging in the top row include qualitative or quantitative empirical data typically used to evaluate a didactical design involving modelling activities in a particular (local) educational context. This context in question is often closely related to the authors' own teaching practice or to collaborations between researchers and teachers. Empirical data in papers placed in the second row have a more global or generic function, such as being used for illustrative purposes to give meaning to theoretical notions or as a basis for developing theory. In addition, pure theoretical based designs without a clear focus on its implementation in teaching practices are also placed in the second row.

Concerning the dived between the two columns it is important to note that the theoretical elements used or developed in the two are different in nature. This divide goes hand in hand with emphasising either modelling as a means for the learning of mathematics, or modelling as an educational aim in its own right. Hence, in the left most column the theoretical elements are typical general theories on the teaching or learning of mathematics, while in the right most column the theories used or developed are related to the area of teaching and learning of mathematical modelling and applications.

In the analysis we found some of the WGAM papers (n=15, 15%) challenging to place in exclusively one of the four categories. Multiple papers (n=6) clearly had a focus related to teaching practices but were hard to place with respect to category (1) and (2). Similar there were theoretical oriented papers (n=9) that posed the same conundrum for category (3) and (4). One type of papers that was categorized as belonging to one of these mixed categories is meta-studies. It should be noted however, that we did not find this ambiguity of categorising the papers with respect to the category-pairs (1) – (3) or (2) – (4), meaning that the contributions either were either empirical in nature focusing on the practices of teaching modelling or theoretical with ambition to build or develop theories. Having said this, we now turn to present the result of our analysis.

## Results: Characterisation of the contributions in each of the four categories

In the following we present a brief description of each of the four categories (1)-(4) with exemplary cases for illustration. In the discussion we pinpoint challenges for research in order to support further the integration of applications and modelling in practices of mathematics teaching.

(1) *Integrating modelling in the teaching of mathematics*

The 34 contributions (33%) categorised as belonging to category (1) often focus on using a modelling approach to provide a learning environment for a specific mathematical content. The scope of the particular content in question however varies from well- and narrow-focused content and areas to one or more general mathematical ideas and constructs. For example, Carriera and Baioa (2015) used experimental activities involving modelling the design of a "convenient" staircase to study how two classes of 14-15-year-old students conceptualize slopes of linear functions. The students went out in the city to experience and collect data (make measurements) of a number of different stairs displaying great diversity both between them and internally (having inhomogeneous measures of the raiser (rise) and the tread (run)). Working and analysing the data the students' developed models for designing the stairs based on a constant overall slope and homogenous steps. Carriera and Baioa found that the students calculated the overall slope by either considering the steps total rise over total tread (run) or averaging over the slopes of the individual steps.

Examples of slightly larger content areas are illustrated by the research by Zell and Beckmann (2010), and Blomhøj and Kjeldsen (2007) respectively. Zell and Beckmann (2010) studied German 6th-grade students' work on one of three physical experiments focusing among other things on what aspects of the concept of variable the experiments elicited. Using a framework conceptualizing variables by Malle (1986) the research concluded that all of the aspects of variable in the framework (e.g. object-, placeholder-, and calculational aspects of a variable and nuances of these) surfaced in the students' work and were articulated in post-experimentation interviews. Although variables were used and expressed mostly on a descriptive level, Zell and Beckmann noted that the students especially noted the functional relationship between two measured quantities and generally concluded that the physical experiment, though cognitive challenging, motivated the students and provides a good venue to elicit and introduce different aspects of the concept of variable.

In the case of Blomhøj and Kjeldsen (2007), drawing on data from a course in mathematical modelling structured around six mini-projects, modelling is used to challenge first year university students' conceptions of integrals. The paper reports on the students' perceptions of the integral concept before and after engaging in working with the mini-project $CO_2$-balance of a lake "designed to challenge students' understanding of the concepts of the definite integral and the antiderivative, the significance of the constant, and the interpretations of these concepts in different problem situations" (p. 2073). Blomhøj and Kjeldsen found that by engaging in the mini-project the students' were challenged and explored their understanding of the connections between derivatives, anti-derivative functions, and definite integrals as well as to demystified the arbitrary constant C in the formula for the anti-derivative in a concrete way. More generally the modelling course as a whole supported the students in developing abilities to apply and interpret these concepts in relation to each other in particular contexts, and that that mathematical modelling provides opportunities for students to learn mathematics in different, non-traditional, ways.

An even more general mathematical idea is the focus of a series of papers by Doerr and colleges (Doerr & O´Neil, 2011; Ärlebäck, Doerr & O´Neil, 2013; Doerr, Ärlebäck & O´Neil, 2013) who report on research using a models and modelling perspective (c.f, Lesh & Doerr, 2003) to design and evaluate teaching and learning of average rate of change as the major structuring theme of a six week summer program for beginning university freshmen students.

As indicated above, some of the contributions in category (1) discuss modelling as a way to elicit and develop more general mathematical concepts and ideas often stressed in mathematics curricula. Grigoras (2010) for example used a task were students explore and investigate a pattern of crates on Mars due to meteor impacts to study 13-14-year-old students' mathematization processes. The students are asked to describe, represent and organise the craters as well as to find potential relationships between the set of craters in order to judge if multiple craters could be due to a single meteor impact. Analysing transcripts of video recordings of the students' group work using the notions of fundamental ideas (c.f. Schweiger, 2006), Grigoras reports that the students repeatedly engaged in *approximation* and *geometrisation* when conceptualizing the craters as more abstract mathematical objects; *locating* and *measuring* when representing the positions and distributions of the craters using coordinate systems; *number/counting* and *optimization* as they organized and investigated the potential pattern of the craters. Grigoras noted that some of these fundamental ideas were implicit in the students' work in the sense that they were not explicitly articulated in the students' discussions and that although elicited, they could not be used or built on productively by the students due to their limitations in previous knowledge and experiences. Somewhat along the same line as Grigoras (2010), Siller, Kuntze, Lerman and Vogl (2011) investigated in what way 159 pre-service teachers from Germany and Austria understand modelling as a fundamental idea. Using an exploratory study design their analysis of the pre-service teachers' answers on a questionnaire initially put modelling as perceived on par with other fundamental ideas such as functional dependence, argumentation/proof, and generalising/specialising. The continued exploration of the data reviled "[however], in the cluster analysis, it became apparent that a large portion of the pre-service teachers saw modelling as relatively insignificant compared to other big ideas". (p. 997).

(2) *Developing, implementing and analysing modelling projects and activities in teaching practice*

The 43 papers (42%) that we categorised as researching empirical aspects of modelling as an educational goal in its on right show great diversity in scope and focus. Some of the contributions study issues related to students' activity, engagement and leaning, whereas other investigate the role of the teacher and teacher behaviour. In addition, on the one hand one can find papers that are inherently empirical and rich on data, and, on the other hand, more methodological contributions, reporting on the design of research instruments and modelling activities. One set of papers address the issue mentioned above regarding that fact that modelling by and large not is part of everyday mathematics classrooms practices as pointed out by Burkhart (2006), Blum, Galbraith and Niss (2007), and Blum (2015).

In this context, and focusing on teachers a key stakeholders, Ärlebäck (2010) presents a case study investigating teachers' beliefs about mathematical models and modelling as understood in terms of a belief structure consisting of the beliefs of five (sub-)belief objects: *the nature of mathematics*; *the real world*; *problem solving*; *school mathematics*; and, *applying, and applications of, mathematics*. Transcribed interviews with two teachers partly centred around five mathematical problems serving as a basis for discussion and reflection (three standard textbook problems, one Fermi Problem, and one modelling-eliciting activity (c.f. Lesh & Doerr, 2003)) are analysed using a contextual sensitive categorization scheme based on the five sub-belief object. The study found that the two teachers did not have any well-formed beliefs about mathematical models and modelling, and that the beliefs structure of the teachers contained inconsistencies which were made explicit within the framework.

Also focusing on teachers' beliefs about models and modelling, the study by Bautista, Wilkerson-Jerde, Tobin and Brizuela (2013) explore the relationship between mathematics teachers' educational backgrounds and their expressed perceptions about mathematical models of a real-world phenomenon as well as the relationship between models and real data. Working with 56 US in-service teachers (grade 5-9) participating in a professional developmental program, a content analysis of written responses to three open-ended questions revealed, similar to the results of Ärlebäck (2010), that the teachers did not hold a unitary understanding of the notion of mathematical models. Further, factoring in the participating teachers' educational background, Bautista et al. (2013) concluded that teachers with backgrounds in Science Disciplines and Mathematics Education tended to tone down the notion of models being exacts and rather stressed the flexible utilitarian aspect of models as tools. Teachers with background in Other Disciplines on the other hand, exhibited a more rigid view of models as either exactly right or completely arbitrary, as well as stressing the importance producing an exact result.

Focusing on another aspects of teachers as key stakeholders, Schmidt (2010), working in the LEMA-project[1], reports on the design and first results from a questionnaire aiming at empirically assess teachers' arguments against and for modelling in terms of obstacles and motives (c.f. Burkhart, 2006). The paper describes the development of the questionnaire, resulting in an instrument containing 120 5-level Likert question organized in 23 categories intended to capture areas where obstacles and motives are likely to surface relative a given *offer-and-use model* (i.e. a conceptualization of how to think about

[1] LEMA stands for *Learning and Education in and through Modelling and Applications* and was a EU Comenius founded collaborative project between Cyprus, Germany, Hungary, France, Spain, and the United Kingdom aiming at producing material for professional development (see http://www.lema-project.org/ ).

the effectiveness of a lesson as dependent of various inputs and influences on the quality of teaching). The research instrument is tested and evaluated using data from 240 teachers, resulting in qualitative rich descriptions of some of the 23 categories of obstacles and motives.

The work presented by Schmidt (2010), as well as her continued work in this area, is followed up by Borromeo Ferri and Blum (2013) who focus on teachers at the primary level. After adapting the final version of the questionnaire from the LEMA study to be suitable for primary teachers, Borromeo Ferri and Blum subjected 71 primary teachers to a questionnaire comprising 43 items distributed over 14 scales and an additional open item. The analysis showed that the three most essential barriers for primary teachers to implement modelling in their mathematics lessons are foremost *lack of material*, followed by *time pressure* (lack of time), and concerns related to *assessment* of modelling.

(3) *Using and developing theories on the learning of mathematics connected to modelling*

In this category containing 6 papers (6%), the focus is on using and developing theory, which can facilitate and support the integration of research on the teaching and learning of modelling in mathematics education research in general. It could be theories on different ecological levels of the educational system, e.g. curriculum level, teacher education, textbook, assessment systems, classroom interactions or students learning. We illustrate the scope of the category by means of two very different examples from CERME 7 and 5 respectively.

In the paper *Modelling in an integrated mathematics and science curriculum: Bridging the divide,* Wake (2011) are combining different theoretical elements form pedagogic, science and mathematics education research; namely critical pedagogic, contextualisation, a focus on key subject matter concepts, problem solving and modelling, in order to form a theoretical framework for understanding the possible roles of modelling in interdisciplinary interplay between mathematics and science teaching. The framework is illustrated by its use in the design and analysis of an interdisciplinary project at upper secondary level on the natural phenomena of floating.

The aim of the paper is to develop a theoretical framework, which can support interdisciplinary teaching in mathematics and science by means of mathematical modelling and applications of models. The goals for the students learning to be persuaded in such interdisciplinary teaching are key concepts in science and mathematics as well as the development of their modelling competency. In this latter sense, the paper is close also to category (4). However, the author himself characterises the research by saying that “This type of exploratory environment, therefore, might be classified as being of the perspective ―Educational modelling type (b) conceptual modelling (focusing on conceptual introduction and learning) in the classification system as proposed by Kaiser et al (2007)” (p. 1007). The modelling activities in the interdisciplinary teaching context are primary seen as a means for the students to communicate with, and make sense of, the science and mathematical concepts involved.

The paper is anchored in the EU-project COMPASS[2] and is hereby representative also for a strong current trend in the research field. During the latest decay EU has launch numerous large research and developmental projects within the field of mathematics and science education. Many of these projects include explicitly or implicitly mathematical modelling and applications and contribute to the field with a large number of developmental projects with designs for teaching and professional development activities as well as with theoretical developments. All these EU projects can be accessed via the web portal www.scientix.eu.

Ruiz, Bosch and Gascón (2007) is an example of a research paper entirely framed within a single theoretical framework, namely ATD. The paper analyses the conditions needed to teach and learn functional-algebraic modelling in an experimental activity designed for and implemented at the end of

[2] COMPASS stands for *Common problem solving strategies as links between Mathematics and Science*

secondary level. In addition, the constraints that hinder the development of such a teaching practice are also analyzed theoretically. The focus is on the theoretical underpinnings within ATD of the didactical design of a modelling situation. The didactical design takes its point of departure in the question of how to earn money from the production and sale of T-shirts. It is discussed in details how to construct a series of praxeologies, which can support the students learning of algebra and functional relationships through modelling activities in this design.

In ATD modelling in, as well as with, mathematics is seen as an essential activity for the learning of mathematics, and as a consequence research with this framework inherently contributes to the development of theories on the learning of mathematics through modelling activities. However, ATD based research are not concerned in particular with the development of the students modelling competence as an aim in its own right. The extra mathematical elements in the modelling process and the students' critical reflections related to the validity of models and/or models role and function in societal contexts are not in focus in ATD. Therefore, typically ATD-based research focuses on modelling as a means for learning mathematics and thus qualifies to belong in category (3).

In relation to the integration of modelling in existing practices of mathematics teaching it is a challenge for the ATD approach that the theory is not easy for teachers to understand and that its implementation requires rather thorough and radical changes in the organization of mathematics teaching.

Only five papers are placed in category (3). Some of the papers in category (1) contain elements of theory for modelling as a means for learning mathematics, e.g. Borromeo Ferri (2007) and Vos (2007). However, our analyses points to a scarcity of research in category (3), which seek to utilize and develop further our theoretical understanding of the potentials and difficulties related to the learning of mathematics by means of modelling.

(4): *Developing theories on the teaching and learning of modelling competency*

Most of the papers in category (2) draw on theory on the teaching and learning of mathematical modelling, often in the form of connected notions and viewpoints developed in the field of research on modelling and application. This includes amongst other things different versions of the modelling process, ideas about the difficulties students might encounter in different sub-processes, and ways of conceptualising progress in the (individual) students' development of modelling competency. Although the theoretical developments in the field were surveyed by Blum (2015), it is characteristic of these theoretical notions and ideas that they are dynamic and still developing and gaining new meaning, while being use in different contexts for research and development of teaching. Therefore, in quite a few cases papers placed in (2) can be said to contribute also to the development of theory. However, in order to be categorised in (4) the main focus of the paper should be development of theory. We have found only four papers (4%) fulfilling this criterion, and we will showcase two of these for illustration.

The paper by Henning and Keune (2005) presented at CERME 4, and later used in the authors' research, is an example of a theoretical contribution regarding the development of modelling competence as an educational aim. In this study the authors develop a model with three levels for characterising the development of the students modelling competence, namely: *Level 1: Recognize and understand modelling*; *Level 2: Independent modelling*; and *Level 3: Meta-reflection on modelling*. Each level is specified in terms of which competences the students should have at that particular level, and it is illustrated through examples of modelling tasks and what types of challenges students should be able to deal with at each competence level.

In the paper by Cabassut (2009) the concept of didactical transposition within ATD is used to analyse mathematical modelling, and in particular focusing on mathematisation in mathematics teaching at the primary level. Through analyses of examples a notion of the mathematisation process as a double transposition connecting the mathematical world and world of real life is developed. "The mathematisation teaching is the place of a double didactic transposition, one from real world into the classroom and the other one from the mathematical world into the classroom." (p. 2157). The theoretical noting of the double transposition in mathematisation capture a fundamental difficulty for the teacher in related to the teaching of mathematical modelling. The notion is used for analysing the PISA model for mathematisation. The research reported is connected to the EU project LEMA for professional development (see Note 1). Accordingly, in the paper, the author proposes generic questions to be addressed in professional development for teacher to help them deal with the challenge of the double transposition: "In a mathematisation task, what knowledge of real world and of mathematical world has to be transposed? What techniques, justifications and validations from both worlds have to be used? How different knowledge, techniques, justifications and validations in the two worlds are articulated and interfering? What effects on teachers' practice, on pupils' learning and on class didactical contract have these articulations and interferences?" (p. 2164).

**Discussion**

Our analysis with respect to the four categories show that over 75% of the WGAM papers are directed towards the development of practices of teaching applications and modelling in various educational contexts rather than on developing theory on the teaching and learning of modelling. The result shows that the papers primarily developing practices of teaching are close to evenly distributed between focusing on modelling and applications as a means for learning mathematics (category (1)) and the development of modelling competency as an educational aim (category (2)). Only about 10% of the WGAM papers have their main focus on using or developing theory. These are evenly distributed between category (3) and (4). 15% of the contributions could not be characterized as belonging uniquely to one of the four categories, and nine out of these 15 papers were theoretical in nature.

Concerning educational levels we only found a few cases directed towards primary education. The vast majority of the papers were found to be focusing on secondary education, with about half categorized belonging to category (1) (primarily papers involving the lower secondary level) and half belonging to category (2) (primarily papers involving the upper secondary level). The around 15% of the papers addressing teaching at tertiary level were found to be evenly distributed in category (1) and (2).

Even though the format and the tradition developed in the WGAMs invites research reports on developmental projects rather than theoretical syntheses, we think that the picture emerging in our analysis is also a reflection of the situation within the research field more generally. The majority of the research in the field internationally is driven by a wish to develop and improve the practices of mathematics teaching by means of integrate applications and modelling.

In many European countries applications and modelling is already part of the mathematics curricula, especially at the secondary level, and in several other countries there are a quest for reforms. Although reforms have been influenced by the research in the field of applications and modelling, there still is a general need for development at curricula level, especially regarding formats of assessment including applications and modelling. This, together with professional development for teachers, is crucial for the integration of applications and modelling in the actual practices of mathematics teaching (Blum, 2015). Therefore, there is a need for research, which can provide a basis for such developments.

It is interesting to notice that in this past decay the EU has strongly promoted and supported a political agenda for the development of mathematics and science education in a direction which emphasizes inquiry based teaching, applications and modelling and the integration of IT. This educational policy is seen as instrumental for the socio-economic development needed Europe as established for instance in so called "Rocard report" by the EU-commission (Rocard et al., 2007). Accordingly, a number of large projects, typically with 6-12 countries involved, focusing on mathematics teachers' professional development have been launched. Several of these projects have applications and modelling in mathematics teaching as part of their programme, and related research and developmental work have been presented at the WGAMs. In fact the WGAMs have also served as a basis for recruiting researchers from different European countries to these projects. Issue related the organisation and the role of research in these projects have been discussed at WGAMs. It is clear that often in these projects, it is difficult to allocated resources for more theoretical research. In line with this, we find that the strong focus on developing the practices of teaching applications and modelling in the WGAMs is both natural and relevant in relation to the European situation as well as the function of WGAM in relation to other activities in the research field internationally.

However, our analysis shows a scarcity of theoretical research in the WGAMs. In particular in category (3) we see a need for research developing theory which can establish connections between the potentials for learning mathematics through modelling and what is known empirically and theoretically explained about the learning of mathematical concepts in general. There are a few cases of theory driven research within the ATD and RME frameworks, but no cases of theory development explicitly addressing this missing connection. The many papers on developmental work supporting the students' learning of mathematics through modelling presented at the WGAMs actually provide a rich and easily accessible empirical material for exactly this type of theory development.

Concerning category (4) we mentioned that the basic theoretical notions in the field are still being explored and gaining further meaning and mutual connections through use and re-contextualisation in concrete teaching and learning situations. In that respect, some of the papers in category (2) are contributing to the development of theory about the teaching and learning of modelling competence. However, despite the few cases placed in category (4), we miss research striving to develop our theoretical understanding of mathematical modelling. Also, here there is potential for using the WGAM papers as an empirical basis for theory development.

In general, we find that the papers presented at the WGAMs represent a rich and multifaceted body of research exhibiting a close connection between the development of practices of teaching applications and modelling at various levels and forms of mathematics teaching. The element of theory and theory development in the coming WGAMs can be strengthened by initiating research using the WGAM papers as a resource for theory development. Hereby, the interplay between research and development of teaching practice can be enhanced further in the continuing work of WGAM.

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