Research on probability and statistics education in ERME: Trends and directions

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Abstract

Our goal in this chapter is to provide an overview of trends and directions within the Probability and Statistics Education Working Group of CERME and to make comparisons with international trends in these areas. In undertaking this task, we focus on the following themes: a historical overview of the working group in CERME; the nature of probability and statistics with an emphasis on connecting the two topics in teaching; the nature of research (descriptive and prescriptive) in the WG; traditions regarding probability and statistics education across European countries; and the role of technology. We conclude the chapter with future directions based on emerging themes, such as modelling, teacher knowledge, professional usage of statistics, effective interventions, assessment, advanced concepts and techniques.

Probability and statistics have a special place in the field of mathematics. They are often considered as sub-disciplines or even, in the case of statistics, as a discipline in its own right. Indeed, statistics is a science related to the political, social and economic history of a country, which explains why there are strong cultural differences in teaching practices. In the 17th century, which is considered to be the dawn of modern statistics, two schools were opposed: the descriptive German naturalistic school and the English political arithmetic which developed treatment and extrapolation techniques based on the growing theory of probability. While the latter has been adopted very quickly by most countries, the strength of the descriptive tradition depended on the country. Today, researchers in mathematics education agree that it is essential to combine a data-centric perspective with a modeling perspective. Nevertheless, this combination takes different aspects from one country to another.

This history is reflected in the working group at CERME that addressed these topics. It was initially called ‘Stochastic Thinking’, to emphasise the interdependency between probability and statistics. However, it turned out that the term stochastics was ambiguous. Where its German equivalent (*Stochastik*) captured this interdependency, in most other languages, stochastic has a very particular meaning, as in “stochastic function” – a function with a random variable. In practice, most papers in the Stochastic Thinking working group focused on either probability or statistics education. For these two reasons, the working group was renamed ‘Probability and Statistics Education’.

This chapter aims to provide an overview of directions and trends within the Probability and Statistics Education Working Group of CERME, but also compare them with international trends in these areas. We start with a brief history of the working group, and discuss the main themes that recurred throughout the years. These include the relation between probability and statistics, technology, teacher knowledge and the need for interventionist research that goes beyond the description of problems but offers suggestions of how to improve probability and statistics education. We end with a wish list for the future.

Brief history of the working group

The Stochastics working group was founded at CERME 3 in Bellaria, Italy. The group has met at each CERME conference since that inaugural year, up to 2015 chaired by Dave Pratt (3), Rolf Biehler (1), Andreas Eichler (1), Arthur Bakker (1) and Corinne Hahn (1). At CERME 3, there were researchers from the USA, Australia and Brazil and, over the years, participants from continents beyond Europe have attended the working group on a regular basis with participants from the USA attending 6 out of 7 conferences and from Australia and Brazil three times. Overall, the largest national contingent has been from Germany (20 researchers) and Spain (18), followed by those of the USA (11), Great Britain (11), Portugal (10) and Greece (10). There has been at least one researcher in the working group from Germany and from Spain at every conference - compared to Greece (6), Great Britain, Italy and France (5). Diversity has been especially strong since 2011, with participants from 12 to 15 different countries in attendance.

The continuity of the group was nearly broken after CERME 6, when only 7 papers were presented, leading to questions being raised within the CERME board about its viability. Nearly catastrophically for the working group, an email to Dave Pratt advising of the intention to disband the group was lost for several months in junk mail. Just in time, when the message was discovered, a campaign was mounted, with huge support from Carmen Batanero, to present two arguments. First, it was suggested that the reduction in size of the working group was a blip rather than a trend (after all, the statistics group was well placed to make such an argument!). Second, and more profoundly, it was argued that the loss of this working group, more than any other, would entail the loss of a unique type of thinking. Drawing conclusions based on data requires attention to context, thinking about aggregates and abductive logic, not typical of other areas of mathematical practice. Since that near extinction, the group has thrived with 22 papers and 10 posters being presented at CERME 9.

At the inaugural meeting in Bellaria, 17 papers were presented across four themes: probabilistic thinking, statistical thinking, teacher education and computer-based tools. These turned out to be recurrent themes throughout the history of the group.

The theme on probabilistic thinking has put forward new theoretical perspectives and evidence that recognised the context-sensitive nature of students' probabilistic thinking. In fact, it has been claimed that, even though by nature probability is more mathematical than statistics, the concept of probability is inherently very complex and very different from other mathematical concepts. There has been concern that the increasing popularity of exploratory data analysis (EDA) has led to the isolation of probability in the curriculum. Below there is discussion on the role that modelling with digital technology might help to reconnect data and chance. However, there was also a suggestion that an increased emphasis on subjective probability might counter the all-pervasive reference to coins, spinners and dice, which are not now so common in children's culture. There has also been research reported on the understanding of risk, which, although ambiguously defined, does carry some connection with probability and might be an interesting domain for the exploration of subjective probability. Further discussion of this theme can be found below.

Concerns have been expressed about negative attitudes in society towards statistics and statistical thinking. These attitudes are similar to those towards mathematics though, even worse, mathematically minded scholars sometimes reject statistical ways of thinking. There has been research in this theme on the role of language as a mediating tool in learning statistics. This research suggests that students may have good intuitions but often not the statistical language to express these. Research has been reported on how people interpret statistical information from authentic contexts such as newspapers. This research raises the question of what can be considered as statistical as well as the question of the authenticity of the activities. Research has tried to tease out the important role that the construction of a task and the subsequent social interaction has on the quality of observed statistically related discussion. What are students' situated understandings of basic concepts such as average, spread, distribution, determinism, causality, randomness, stochastic and physical independence? This has led to a major theme around the role of context in statistical thinking.

In fact, the role of context is very different in statistics from in mathematics. Mathematics as a discipline typically aims to be decontextualised whereas statistics typically draws on context. In the mathematics education literature, contexts in word problems are reported to present children with additional difficulties, whereas in statistics the contextual interpretation is important. Nevertheless, if tasks lack authenticity by providing students with an artificial context, students are likely to bring in personal knowledge that is not necessarily statistical. Rather than thinking of abstracting, at least in statistics and probability, as a process of decontextualisation, a focus on enriching, disciplining and refining seems to place emphasis on abstracting as a process of generalising. Research on statistical thinking is further elaborated below.

In the theme focused on teachers, CERME research papers often report on the impoverished nature of training for teachers of statistics who were not especially knowledgeable in that area. Questions have been raised about how the community might support teacher development in using innovative pedagogies and to become more connectionist in their approach. The key influence of the methodology of teachers on the learning of probability and statistics has been noted. There has been concern that, while research had been finding evidence about what teachers did not know, to design effective teacher education, research would need to identify what teachers know already, including their attitudes, and what they need to know to be effective teachers. One striking observation was that there are many theoretical frameworks for teacher knowledge and so there is a need to clarify the different emphases in the different frameworks and over time reach some convergence in terminology. Details of the teacher education theme are discussed below.

CERME research papers have reported on the importance of computer-based tools in the teaching and learning of probability and statistics, for example in potential for students to appreciate probability distribution as an emergent phenomenon and key concepts such as the mean. Design-related questions have been raised about how research might identify significant affordances of computer-based tools to realise such potential, including the role of microworlds. It has been proposed that digital tools can offer a pathway towards the effective use of modelling to re-connect probability and statistics. Most recently, there was a recognition that the design of learning environments needs to consider the use of computer-based tools alongside the design of the task itself and the nature of classroom interactions.

There is a tendency for some topics to be reinvented by each generation (e.g., misconceptions research), possibly emphasized by inclusive nature of CERME. So, even if some trends can be identified, they lie within a maelstrom of repeated research, which does not always build on prior results. Another tendency is that CERME papers follow international trends, such as emerging interest in inference or sampling. The Forum for Statistical Reasoning, Thinking and Literacy (SRTL) is one of the international influences in this respect, just like the conferences of the International Association for Statistical Education (IASE) such as the International Conference on Teaching Statistics (ICOTS) .

The nature of probability and statistics

Probability and statistics have very different historical origins (Stigler, 1986). We see this reflected in how the topics are taught: Probability is highly mathematical (based on combinatorics) while statistics is multidisciplinary. Statistics education is a marginal discipline in the sense that it is at the boundary of many other disciplines including mathematics education, statistics and psychology (Groth, 2015). Statistics education research is carried out by very different groups of researchers, who publish in a diversity of journals. The fragmented nature of the field also has an upshot. Statisticians can play in everyone else's backyards. Statistics educators typically draw on many different fields. For a long time, statistics education has lain behind in terms of theoretical and methodological rigour, but is catching up with, say, mathematics education (Nilsson, Schindler, & Bakker, in press).

In CERME 3, where the Stochastic Thinking working group was first introduced, the papers presented in the group tended to focus on statistics and probability as separate topics. However some studies (Nilsson, 2003; Pratt, 2003) brought out the importance of providing students with an experimental situation and computer tools that can help them experience the dual notion of probability (epistemic/subjective and statistical/frequentist) (Hacking, 1975). In the following meetings, there were several studies addressing this idea of experimental learning environments with technology.

Abrahamson and Wilensky (2005) reported on a study with 26 8th-grade students conducting probability experiments using NetLogo models that randomly generate blocks of 3x3 arrays red and green squares and accumulate the outcomes in columns according to the number of red squares in each block. Researchers designed the task in a way that promoted students’ understanding of the connection between the distributions of empirical outcomes from small samples they collected using NetLogo and the distribution of the combined empirical outcomes from all small samples in a collaborative learning environment. Students’ probabilistic reasoning was supported by their analysis of distribution of empirical outcomes in this experimental approach to probability.

Prodromou (2007) investigated 15-16-year-old students' coordination of data-centric distribution and modelling distribution as they worked in a microworld environment about throwing basketballs where they could use causality to articulate features of distribution. The paper focussed on the work of two pairs of students. Although these students seemed to intuitively understand that the data-centric distribution would converge to the modelling distribution, which was the intended outcome, they had a difficulty in viewing the modelling distribution as the generator of the data-centric distribution.

In Schnell’s (2013) study with students aged 11 to 13, the focus was on the random data generated from chance experiments using a computer simulation tool and identifying the patterns and variability in them in short term and long term contexts.

The emphasis on both the frequentist approach and the classical approach to probability in school mathematics curricula also stimulated the discussion of new approaches to teaching probability with the development of new technology. For example, at CERME 7, Henry and Parzysz (2011) provided perspectives on the use of computer simulations for linking the frequentist and the classical approaches to probability given the emphasis on teaching both in French high schools. They argued that the use of computer simulations in the classroom as a pseudo-random generator would help learners develop a better understanding of the statistical and probabilistic ideas, such as relative frequency and variability.

In addition to the frequentist and classical approaches to probability, subjective interpretation is also important and even more intuitive for students when teaching probability. In more recent years, research focused on combining subjective ideas of students with the empirical data from random experiments. For instance, Helmerich (2015) studied 8-10 year old children's use of subjective ideas and empirical data from experimenting with different "odd dice" in a game context. Moreover, Kazak (2015) reported on 10-11 year old students' coordination of their subjective ideas and the empirical data in attempt to evaluate the fairness of a chance game. Students also expressed degrees of confidence as they played the game and used different amounts of data generated by TinkerPlots simulations.

Besides the use of computer simulations, there are other studies suggesting effective use of another pedagogical approach to make the connection between theoretical and empirical probabilities in connection with experimental approach to probability. For example, enacting of the given situation, such as flipping a coin (Diaz-Rojas & Soto-Andrade, 2015) and frog jump (Eichler & Vogel, 2015), tended to promote students' statistical understanding.

Different from the focus of aforementioned studies on the data-centric perspective (or experimental approach) in teaching probability, Ben-Zvi, Makar, Bakker, and Aridor (2011) brought out the notion of probability within informal statistical inference. They reported on a study of 11-year-old children's reasoning about sampling when making informal statistical inferences in an inquiry-based environment. Engaging students in making informal statistical inferences from samples allowed them discuss the notions of likelihood, level of confidence and randomness together with the statistical notions, such as distribution, spread, average.

With the increasing attention to informal statistical inference at school level, research discussed in the more recent CERME meetings tended to focus on the notion of probability in the context of informal statistical inference and informal inferential reasoning. For instance, Jacob and Doerr (2013) presented their study about secondary students' informal inferential reasoning as they collected a sample of data in an attempt to draw a conclusion based on the related sampling distribution supported by the use of Fathom software. This study pointed out the importance of probabilistic ideas, such as level of uncertainty and law of large numbers, in making a sound connection between samples and the sampling distribution. Henriques and Oliveira (2015) also studied 8th grade students' informal statistical inference during statistical investigation involving body measurements of students at the school. Students analysed their data using TinkerPlots software. When expressing the uncertainty to make generalizations beyond data, students tended to use probabilistic language, such as "probably", "maybe", "something similar" and "tend to be".

Role of context

Already in 2003, authors raised the question of the effect of context (Monteiro & Ainley, 2003). At CERME6, Eichler asked the question explicitly ("the role of context in stochastic education"). It became particularly pregnant in CERME7 where it was the subject of important discussions.

Several authors have raised the question of the authenticity of the problems proposed to pupils, a question explored in mathematics education since the 1980s, but of particular importance here, as the context is an integral part of Stochastics, interdisciplinary by nature. This is even more prevalent since the rise of the EDA perspective (Borovcnik, 2005). Researchers who explored the role of context in the learning of statistics (see e.g., *Educational Studies in Mathematics*, 45(1), 2001; *Mathematical Thinking and Learning* 13(1&2), 2011) usually recommend the use of real data. Nevertheless, they identified that this use could be a problem, particularly because of the difficulty experienced by students to extricate themselves from the context and the weight of their personal beliefs.

What could an authentic situation look like? We identify two main trends through CERME proceedings: Some researchers recommend the implementation of an inquiry-based process where students collect and handle their own data. Other researchers propose activities based on the study of information given by the media. Within the first trend, Ainley and colleagues (2011) designed and experimented a sequence of science activities which focus on different aspects of exploring flight. Students were led to explore variability through the repeating of measurements. Eckert and Nilsson (2013) conducted an experiment based on farming experience plans: They stimulated students to think about the variability of the results of their pumpkins and sunflowers plantations. Hauge (2013) proposed to build school activities about risk assessment by drawing inspiration from the management of wild capture fisheries. Within the other trend, Sturm and Eichler (2015) used HIV rapid test information to make students work on Bayes formula; Plicht and colleagues (2015) studied students' interpretation of graphs about milk production.

Technology

In this section, we discuss how research reported by technologically oriented papers in the CERME conference proceedings has connected to the ideas raised in the handbook chapter by Biehler et al. (2013) on how digital technologies are enhancing statistical reasoning at school level. Biehler et al. (2013) described the requirements of digital tools in ways which resonate with an old analysis by a UK quango of the entitlements offered to students by technology (BECTA, 2000). These entitlements consisted of 'learning from feedback', 'observing patterns', 'seeing connections', 'working with dynamic images', 'exploring data' and 'teaching the computer'. The requirements laid out by Biehler et al. (2013) included a capacity for students not only to practice graphical and data numerical data analysis, engaging in 'exploring data' and 'working with dynamic images' but also to create new methods, such as by programming or similar activity involving 'teaching the computer'. Biehler et al. (2013) also recognised the importance of using embedded microworlds, thus 'learning from feedback', and constructing models, which would also involve 'teaching the computer'. It is not difficult to see how 'observing patterns' and 'making connections' are fundamental to all of the requirements of tools as described by Biehler et al. (2013).

Amongst the main themes identified by Biehler et al. (2013) was the issue of when students should use software as opposed to working manually. Without exception the technology-oriented papers in CERME proceedings have involved direct use of technology as a tool for learning by students and there has not been research specifically addressing when the use of technology may be beneficial.

A second theme pointed to the tension between adopting technology that might be beneficial for learning statistics and probability but which requires considerable effort to master before such learning might become evident. There is of course an argument that the length of time needed to learn a new technology is time that could have been spent making sense of difficult statistical and probabilistic ideas. It is the teacher under pressure to cover a large curriculum who has to manage this tension (and at school level it is often the mathematics teacher working with a curriculum that contains relatively little content of statistics and probability). Researchers compete for scarce research funds and so encounter a similar tension because they are unlikely to afford the time for the gradual integration of technology into a classroom. Hence, research using software that is more narrowly focussed can be easier to manage than research on the use by students of more general educational softwares such as TinkerPlots and Fathom. Nevertheless, over the years, CERME has reported examples of both.

There have been five studies which have used specially designed microworlds to study student learning of specific key concepts: ‘fairness’ (Paparistodemou & Noss, 2003); the Law of Large Numbers (Paparistodemou, 2005); randomness (Pratt & Prodromou, 2005) and distribution (Prodromou, 2007; Prodromou & Pratt, 2009). These studies demonstrated how digital resources can be harnessed to explore specific research questions, throwing light upon students’ understanding of these key concepts. Reasoning with key concepts is a theme identified by Biehler et al. (2013) but, whereas that report discussed the use of technology in the teaching and learning of key concepts, these CERME studies illustrate the use of technology to research perturbations in students’ statistical reasoning about key concepts. Indeed, at a higher level of abstraction, one other study, Pratt (2003) deployed a microworld to propose a general theory for how probabilistic knowledge emerges. The designed microworlds in these studies were sufficiently narrowly focussed on the specific concept in question that the data collection could take place over a relatively short time span without a considerable time commitment for the students to learn the tool.

Most of the other technologically oriented studies reported in CERME conferences used more general educational software, such as NetLogo, Fathom and TinkerPlots. These studies needed strategies for embracing the challenge of enabling students to master that software sufficiently to elaborate their research aims and so were either closely related to the researchers’ teaching activity or were part of a wider long term study.

Two studies by Abrahamson and Wilensky (2003; 2005) deployed the programming language NetLogo to explore how students’ learned through design and collaboration. In the terminology of Biehler et al. (2013), these studies immersed the students in a setting where they might create their own methods of solution, such as building for themselves the Normal distribution and engaging in collaborative activity around the Law of Large Numbers, thus emphasising the development of aggregate thinking, an important theme in the Biehler et al. (2013) report.

There has been an increasing emphasis on studies involving model construction (1 out of the 2 technologically oriented papers in each of CERME 5, 7 and 8 and all four in CERME 9). These studies have focussed on student’s reasoning with key concepts and aggregates as they: follow a schema for simulation (Maxara & Biehler, 2007); reason informally about sampling using TinkerPlots (Ben-Zvi et al., 2011; Martins et al., 2015); reason about probability and randomization tests using TinkerPlots (Frischemeier & Biehler, 2013); compare groups (Frischemeier & Biehler, 2015); reason about uncertainty while playing a game and using a TinkerPlots simulation (Kazak, 2015) and use simulations for informal inference (Lee et al., 2015).

The move from research that studies either probability or statistics exclusively to studies of these two connected domains in an integrated way has been discussed in the earlier section on the nature of statistics and probability. It is worth mentioning here that this transition is reflected in the changing nature of those studies involving technology. Indeed, it is a reasonable conjecture that the innovation of educational software such as TinkerPlots and Fathom has been a significant trigger for that change. At one time, experimental studies of probability were restricted in the main to students using familiar random generating devices such as coins, urns, dice and playing cards. Some studies then began to exploit additional affordances of technology by simulating those devices. At the same time, early uses of technology focussed on exploratory data analysis, arguably intentionally avoiding the difficulty of probabilistic thinking. Innovations in educational software have facilitated the re-connection of statistics and probability in research studies. The integration into TinkerPlots of samplers that allow the simulation of familiar random devices into exploratory data analysis software marks the move among CERME researchers to conduct research that considers both key concepts and aggregate thinking and allows students to create new methods and models. Without doubt, such research makes additional demands because of the increased time commitment for the students to learn such tools. There is a danger perhaps that the pressure to conduct research with these tools will lead to a narrowing of research settings to those few situations where long-term research is being conducted or where the subjects of the research are in fact closely connected to the researchers, for example through the teaching commitments of the researchers. There continues therefore to be a place for research which is more narrowly focussed alongside the exciting research now being conducted with larger educational software packages.

Teacher knowledge

As the statistics and probability topics have become part of the mainstream mathematics curricula in various countries since late 1990s, teachers' knowledge on these topics (i.e. their conceptions of statistical and probabilistic concepts and ideas) became of an ongoing interest to mathematics education researchers. Discussions about the following main issues related to teachers began in CERME 3 and seemed to be still relevant: (1) impact of teachers' strong beliefs about the nature of mathematics on teaching and learning of statistics (deterministic vs. uncertainty), (2) insufficient training of teachers both in terms of content knowledge and pedagogical knowledge related to statistics and probability.

Much of the research with pre-service and in-service teachers have focussed on teacher knowledge of statistics and probability, involving conceptions, competencies and reasoning in various topics: Probability (Maury & Nabbout, 2005; Peard, 2005; Andrà, 2011; Chernoff, 2011;Contreras, Batanero, Díaz, & Fernandes, 2011; Prodromou, 2011; Eckert & Nilsson, 2013), randomness (Paparistodemou, Potari, & Pitta, 2007), both statistics and probability (Eichler, 2007), graphs (Batanero, Arteaga, & Ruiz, 2009; Arteaga & Batanero, 2011; González Astudillo & Pinto Sosarisk, 2011; Arteaga, Batanero, Cañadas & Contreras, 2013), risk (Pratt, Levinson, Kent & Yogui, 2011), sampling distribution (Doerr & Jacob, 2011), statistical literacy (Koleza & Kontogianni, 2013), uncertainty (Frischemeier & Biehler, 2013; Gonzales, 2015), variability and sampling variability (Gonzales, 2013; Jacob, Lee, Tran, & Doerr, 2015), and measures of central tendency (Santos & De Ponte 2013). A general implication from these studies seems to be the need for improvement of teachers' knowledge of specific content that they are expected to teach.

Having adequate knowledge of content is solely not sufficient for developing students' understanding of statistical concepts and procedures. For instance, Eichler (2007) pointed out that students' difficulties in understanding independence, conditional probability, and Bayesian theorem might be due to the teacher's use of tree diagram in a traditional way, rather than with natural frequencies. This finding suggests the importance of teacher's pedagogical content knowledge which involves “an understanding of what makes the learning of specific topics easy or difficult” (Shulman, 1986, p. 9) and links content and pedagogy related aspects of knowledge for teachers. However, only five of the CERME papers focussing on teachers’ knowledge tended to deal with pedagogical content knowledge of teachers while there were twenty three papers on teachers’ knowledge/conceptions in statistics and probability. For example, one of the studies conducted with pre-service teachers (Paparistodemou, Potari, & Pitta, 2007) indicated some difficulties in combining pedagogical practices (e.g., group work, use of concrete tools, games and time management) and the mathematical content in their teaching and in considering students’ intuitive ideas, possible student responses and how they would think during the implementation of their lesson plans on the idea of randomness for children of ages 4-5.5. Given that these pre-service teacher were doing their teaching practice in pre-primary schools and completed courses on statistics and probability as well as teaching mathematics, there is need for addressing pedagogical content knowledge of teachers in teaching statistics and probability in teacher education programmes. The other four papers focussed on in-service teachers knowledge, in particular pedagogical content knowledge, with regard to teaching topics, like statistical graphs (González Astudillo & Pinto Sosa, 2011), probability (Eckert & Nilsson, 2013), and variability (González, 2013; Quintas, Oliveira, & Tomás Ferreir, 2013) at different grade levels, from primary school to university. Although these studies tended to report on findings from a very small number of teachers, their attempt to identify characteristics of teachers’ pedagogical content knowledge or knowledge for teaching of different statistical and probabilistic topics seem to be promising for our understanding of what knowledge teachers should have in order to promote students’ understanding of these topics.

The use of technology within this theme also seems to be getting more attention in more recent years as discussed in the section on Technology. In particular, training of future mathematics teachers tend to focus on the use of technology tools in the context of modelling a random experiment within Fathom (Maxara & Biehler, 2007) and reasoning about uncertainty during randomization tests with TinkerPlots (Frischemeier & Biehler, 2013). Research with in-service teachers included teachers’ understanding of sampling distribution with the use of Fathom software (Doerr & Jacob, 2011) and teachers’ models of simulation processes in the context of informal statistical inference through the use of TinkerPlots software (Lee, Tran, & Nickel, 2015).

Types of research conducted and needed

What types of research is needed to improve probability and statistics education? A first type of studies is to do a problem analysis, baseline studies, or a needs analysis. Such studies are typically descriptive or evaluative in nature and focus on education as it currently is – mostly focusing on students or teachers.

A second type of studies is to identify sensible learning goals. These could include discussions of statistical literacy. In such cases, scholars analyse what would be good learning goals given today’s or tomorrow’s society. Sometimes further analysis of such learning goals is required. This holds, for example for the concept of risk, which is argued to be societally relevant but also needs further elaboration.

A third type of studies offers suggestions or advice on how to promote particular learning. These are typically design-based interventions. New technology is often used to foster desirable ways of learning (see the section on technology). But there are also creative ideas such as using random walks (Soto-Andrade, 2013)

A fourth type of studies, effect studies and evaluations of interventions, is closely connected to the third: What was actually learned? A closely related question is how to assess student or teacher knowledge or skills validly and reliably.

There has always been much descriptive research in the group, typically about students' statistical or probabilistic knowledge, or lack of it. Such studies are important, for example to flag up a problem in a country and underpin the need for improvement or redesign of the curriculum. At CERME, however, several commentators (e.g., Per Nilsson in 2013) have observed that the research community knows pretty well what the problems are so that we need more design-based, prescriptive or advisory research: Didactical ideas about how to improve probability and statistics education. This implies that in their view, the field asks for more research of the third type. However, to know how effective and efficient these approaches are we also need more systematic evaluation of new interventions.

Future directions

We end with a wish list:

- A large proportion of the papers focused on student learning. But because most mathematics teachers have little knowledge of statistics, more research on teaching is needed. Many teachers try to teach statistics like other mathematical topics, focusing on only the results, procedures, graphs etc., rather than statistical thinking and reasoning processes. What is it that teachers need to know? The concept of Statistical Knowledge for Teaching (SKT) may be fruitful here (Groth, 2007). And even more relevant: How can teachers be supported to develop this SKT?
- In line with the previous point, teachers also need better familiarity with how to use technology. The notion of Technological Pedagogical Content Knowledge (TPACK) has been suggested as a theoretical lens on this issue.
- In mathematics and science education, modelling is coming up as an important learning goal but also a means of supporting learning. As some of the CERME papers indicated, modelling can also as a bridge between statistics and probability in an era when probability is becoming isolated. Technology offers new possibilities, as numerous CERME papers have shown (cf. difference between dice games and computer games), but what are effective ways to promote students' understanding?
- The rare studies on vocational and professional usage of statistics emphasize that statistics in its many contextual manifestations is becoming more and more important for the workplace. More interest from educational researchers for this domain is welcome.
- A large proportion of CERME papers focused on rather basic probability and statistics. Most welcome is attention to more difficult concepts and techniques.

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