

12. Classroom practice and teachers' knowledge, beliefs, and identity

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Issues pertaining to teachers and teaching have gained ever-increasing attention in mathematics education over the last few decades. While teachers and teaching were conspicuously absent in mathematics education research until the mid-1980s (Shulman, 1986), Sfard (2005) reported on behalf of a research team for ICME X that “the last few years have been the era of the teacher as the almost uncontested focus of researchers’ attention” (p. 409).

These issues seem to have gained even further momentum since Sfard and the survey team conducted their study, not least in Europe. At the CERME conferences, they have been dealt with under several headlines. Initially there was one topic working group (TWG), *From a Study of Teaching Practices to Issues in Teacher Education*. This TWG has now been split into several others, and issues related to teachers, teaching and teacher education were at the last two CERMEs dealt with in three TWGs, named *Mathematics teacher education and professional development*, *Mathematics teachers and classroom practices*, and *Mathematics teachers’ knowledge, beliefs, and identity*. In this chapter, we deal with research on the last two of these themes presented at the first 9 CERMEs. We leave issues pertaining to the first theme to chapter 13.

There has been both continuity and change in the ways in which issues of teachers and teaching have been dealt with at the CERME conferences. One continuous effort has been to discuss what a professionalization of teaching entails. In particular, it has been a persistent effort to develop understandings of how teachers contribute to classroom practice and of the types of knowledge teachers need in instruction. Issues related to teachers’ beliefs and professional identity have also played a recurrent but smaller role. At the same time, the field has developed and changed over the last 20 years. Changes are apparent in shifting emphases among the themes addressed as well as in the dominant theoretical frameworks.

In sections 2 to 5 below, we outline developments in research concerning the most significant thematic issues in the field. These are the ones of teachers’ knowledge, beliefs, and professional identity and of their relation to how teachers engage in classroom practice. At CERMEs issues related to beliefs and identity are often dealt with in the TWG on affect (cf. chapter 10), and we limit our discussion to papers presented in the TWGs on teachers and teaching. Based

on our reading of papers presented in these TWGs, we argue that there has been some change within research on each of the thematic issues mentioned, in the relative significance attached to each of them, and in the frameworks that are used to study them. In our concluding section, we argue that there seems to have been a move towards less causal and more dynamic interpretations of the relationships between knowledge, beliefs and identity on the one hand and classroom practice on the other.

Before engaging in the main discussion, however, we introduce what appears to be a dominant vision of school mathematics underlying most papers presented at these working groups.

1. THE VISION OF MATHEMATICS TEACHING AND LEARNING

The vision of school mathematics that orients the larger part of studies on teachers and teaching at CERMEs is to some extent informed by reform efforts that have also been promoted on the other side of the Atlantic (e.g. NCTM, 2000). Studies presented at CERMEs often seek to understand or promote instruction and classroom practice that emphasise mathematical processes, and, explicitly or implicitly, they ask what it takes for teachers to engage students in for instance mathematical investigations and problem solving.

It seems, then, that contributions to CERMEs on teachers and teaching are often informed by what is sometimes referred to as the reform in mathematics education. This vague and ambiguous term refers to developments that draw on constructivist as well as sociocultural approaches to learning and on a view of mathematics as an ever-evolving human construction. Consequently, the reform promotes a vision of school mathematics that focuses on students' creative engagement in exploratory and problem solving activities as they develop their understandings of significant mathematical concepts and procedures. In a draft to the *Principles and Standards for School Mathematics* (NCTM, 2000), the role of the teacher is outlined as follows:

Curricular frameworks and guides, instructional materials, and lesson plans are only the first elements needed to help students learn important mathematics well. Teachers must balance purposeful, planned classroom teaching with the ongoing decision-making that can lead the teacher and the class into unanticipated territory from an effective mathematical and pedagogical knowledge base (NCTM, 1998, p. 33).

According to such principles, the teacher is expected to play a decisive role that goes beyond a tradition of presenting ready-made concepts and procedures for the students to follow and copy.

References to Standards 2000 and other documents in the same tradition (e.g. NCTM, 1989, 1991) are recurrent in CERME proceedings, and, even when no such reference is made, inspiration is often apparent from what Ellis and Berry

(2005) call the cognitive-cultural paradigm in mathematics education. A question underlying many of the papers presented at CERMEs is therefore what it takes in terms of teachers' knowledge, beliefs, and identity for them to support classroom practices in line with those promoted by the reform.

2. STUDIES FOCUSING ON TEACHERS' KNOWLEDGE

Teachers' knowledge has been a dominant theme in educational research for decades. The dominant approaches in mathematics education all draw on the work of Shulman and his colleagues from the 1980s, in which they pointed to – among other types of knowledge – to aspects particularly related to the contents.

Discussions of mathematics teachers' knowledge have been prevalent in every CERME conference. Already in CERME 1, Carrillo, Coriat and Oliveira (1998) discussed mathematics teacher knowledge. They emphasised how this knowledge develops from practice, and they called for a way of talking about knowledge that is gained from experience. Drawing heavily upon the legacy of Shulman (1986), Carrillo et al. described different attempts to characterise types and components of mathematics teacher knowledge, and they discussed the challenging connections between teachers' knowledge and their performance in the classroom.

It is a continuous discussion how best to characterise knowledge that is relevant and specific to the teaching of mathematics, and different approaches have been suggested. In the following, we highlight two major frameworks that have been frequently used in CERME papers on teacher knowledge. Both frameworks rely upon the work of Shulman (1986), but they adopt different approaches to characterising the mathematical knowledge that is specific to the professional work that teachers do in the classroom.

The first major approach to conceptualising mathematical knowledge specific to teaching developed from a study in England referred to as Subject Knowledge In Mathematics (SKIMA). At CERME 3, Rowland, Thwaites and Huckstep (2003) presented a paper that reported on an investigation of prospective teachers' use of knowledge in classroom teaching. From a grounded approach to analysing classroom videos, these authors developed codes that described the actions of the prospective mathematics teachers as they engaged with their students in the classroom. Based on these analyses, they developed the knowledge quartet (KQ), a framework consisting of the four components of *foundation*, *transformation*, *connection*, and *contingency*. Foundation is knowledge and beliefs “acquired in the academy as preparation (intentionally or otherwise) for [teachers'] role in the classroom” (<http://www.knowledgequartet.org/category/foundation/>). Transformation is the capacity to turn foundational knowledge into a form that is pedagogically relevant. Connection concerns establishing coherence among topics dealt with within and across lessons and instructional sequences. Finally, contingency is

about a teacher's readiness to respond flexibly to unforeseen questions, comments, and suggestions from the students. Huckstep, Rowland and Thwaites (2005) presented this newly developed framework at CERME 4, tracing its origin back to their paper from the previous CERME conference. They described the four core units of the KQ, and used it in their analysis of the teaching of a prospective elementary teacher.

The knowledge quartet was initially developed to qualify communication among prospective teachers, teacher educators, and school based mentors during school-based teacher education. In this sense, it was considered a tool for teacher education or teacher development. The framework has also been used as an analytical tool for interpreting prospective and practising teachers' engagement with classroom practice. The fourth element of the quartet, that of contingency, has an emphasis on *knowledge in interaction* and may be regarded as an important contribution to understanding the dynamic character of classroom processes (Huckstep, Rowland, & Thwaites, 2005). Somewhat similarly, Ainley and Luntley (2005) refer to attention-dependent knowledge as both highly contextual and to some extent generalizable, but not as an expression of the application of general rules in a particular situation.

In subsequent CERME conferences, the KQ framework has been applied in a steady stream of studies from different European contexts. At CERME 5, Corcoran (2007) used the framework to analyse classroom data of prospective elementary teachers in Ireland, whereas Turner (2007) investigated a teacher's development of the foundation dimension after having used the framework for reflecting on mathematics lessons over time. Petrou (2009) presented a paper at CERME 6 on adapting the KQ framework for use in Cypriot mathematics classrooms, and he suggested that the framework should be adapted to include teachers' interpretation of mathematics textbooks. More generally, he maintained that adaptation of the framework for use in other countries requires careful attention to possible cultural differences. Rowland, Jared and Thwaites (2011) showed that the KQ, originally developed for use in the education of elementary teachers, is also useful for the analysis of teaching in secondary mathematics classrooms. Also at CERME 7, Turner (2011) returned to an investigation of one of Shulman's original types of teacher knowledge: propositional knowledge. She used the KQ as a framework for investigating differences between elementary teachers' propositional knowledge and knowledge in practice, and suggested that mathematics teachers' propositional knowledge is activated by reflections on practice.

The second major approach to conceptualising mathematics teacher knowledge also developed from Shulman's (1986) ideas about professional teacher knowledge. Like Rowland and colleagues, Ball and her colleagues also developed their practice-based theory of mathematical knowledge for teaching (MKT) from careful analyses of video recordings of mathematics classrooms,

this time from the USA. Whereas Rowland and colleagues decomposed the challenges of teaching practice into categories that were eventually synthesized into the four core components of foundation, transformation, connection and contingency, Ball and colleagues decomposed the work of teaching mathematics into mathematical tasks of teaching and defined MKT as the knowledge needed to carry out these tasks (Ball, Thames, & Phelps, 2008). They then extended Shulman's categories of subject matter knowledge, pedagogical content knowledge, and curriculum knowledge into six subcategories of MKT, two of which have attracted particular attention at CERMEs. These are the ones of *horizon content knowledge* and *specialized content knowledge*. Horizon content knowledge concerns "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball, Thames, & Phelps, 2008, p. 403); specialized content knowledge is knowledge of the contents per se that is special to the profession, that is, knowledge of mathematics that is in some sense unrelated to children and learning, yet essential to the work of teaching.

When Ball et al.'s (2008) seminal article on MKT was still in publication, Ribeiro, Monteiro and Carrillo (2009) presented the main aspects of the framework in a paper at CERME 6. At the same conference, Stylianides and Stylianides (2009) also presented a paper that described some early versions of the framework. From CERME 7 and onwards, Ball et al.'s MKT framework has attracted much attention in the European research community, and a growing number of papers refer to studies that build on it. One line of study has investigated and discussed subcategories of MKT. For instance, Fernández, Figueiras, Deulofeu and Martínez (2011) discussed horizon content knowledge and suggested to re-define this category of MKT so as to relate to the work of teaching. At the next CERME, Jakobsen, Thames and Ribeiro (2013) continued this discussion, but highlighted the importance and usefulness of advanced mathematical knowledge in their working definition of this subcategory of MKT.

Another line of study has focused on specialised content knowledge for teaching mathematics. For instance, Aslan-Tutak and Gunes Ertas (2013) discussed what kind of tasks that can be applied to develop specialized content knowledge in prospective teacher education. Also at CERME 8, Flores, Escudero and Carrillo (2013) argued from their theoretical review that specialised content knowledge is inextricably connected with teaching. They thereby challenged the definition of Ball et al. (2008) that presents specialised content knowledge as a mathematical knowledge that does not require any knowledge or experience of teaching or students. Other studies have investigated specialized content knowledge in different topics or contexts. For instance, Pino-Fan, Godino, Font and Castro (2013) presented a study on prospective teachers' specialised content knowledge for the teaching of the derivative, and they suggested distinguishing

between the application level and the identification level. Zembat (2013) also focused on specialised content knowledge in his attempt to investigate the gap between teachers' current knowledge and the knowledge required to teach mathematics in the United Arab Emirates.

Although authors frequently refer to the frameworks of Shulman and Ball, they tend to supplement these frameworks or modify the perspectives involved. Such adjustments generally emphasize the dynamic character of classroom interaction and the complexities involved as contingencies arise. In the former case, reference is often made to some form of reflective practice or teacher inquiry. This is so as reflection in or on practice is considered an important prerequisite for supporting students' learning in line with reform recommendations as well as a significant vehicle for professional learning. Also in line with reform recommendations, the notion of inquiry generally refers to the activity of both students and teachers. In other cases, different types of knowledge or knowing are pointed to as necessary for teachers, if they are to flexibly orchestrate classroom interaction in line with the visions of teaching outlined earlier. More recently, some CERME papers have shifted their attention from discussing the content and definition of subcategories of MKT to focus on how this mathematical knowledge is embedded in teaching practice. For instance, Kwon (2015) analysed the work of teaching mathematics in a study that evolved from the context of MKT at the University of Michigan. A similar emphasis on teaching practice can also be found in other CERME papers that apply the MKT framework and call for a participationist approach (e.g. Mosvold, 2015). There seems, then, to be the beginning of a trend to shift the focus from knowledge to practice.

Although MKT and KQ are by far the two most frequently used frameworks for studying of mathematics teacher knowledge in the history of ERME, not all studies in this field apply these frameworks. For instance, Kuzniak & Houdement (2001) investigated the development of prospective teachers' mathematical knowledge in the French context. They emphasised "didactical provocation" as a tool for supporting the development of combined knowledge among prospective teachers. In another study, Regecová and Slavícková (2011) applied Brousseau's theory of didactical situations in their analysis of secondary teachers' development of mathematical content knowledge in Slovakia. Other studies developed their own theoretical frameworks, like Canavarro (2011) did in her study of developing mathematics teachers' professional knowledge by use of investigative tasks.

3. STUDIES FOCUSING ON TEACHERS' BELIEFS

The character, development, and possible impact of teachers' beliefs have been recurrent themes in research on and with teachers for decades, and they are also important in studies presented at CERMEs. In many papers, however, the notion of belief is used merely in passing and as an unproblematic reference to

comments made by the teacher in question. In what follows, we focus the discussion of beliefs on the approach in papers with more explicit focus on beliefs or on one or more of the terms that are considered close proxies to one of beliefs, that is, the ones of conceptions, perceptions, values, or orientations.

The original rationale of belief research in mathematics education is the expectation that beliefs serve as an explanatory principle for practice or at least as a significant determiner of what and how mathematics is taught and learnt. Indeed, without the expectation of belief impact it is unlikely that the field would have attracted more than minimal attention. This expectation is also apparent in papers presented at CERMEs. In his introduction to the section on teacher education and teachers' beliefs at CERME 1, da Ponte (1998) contends that "to have some insight into the way teachers understand and carry out their job one needs to know their conceptions and beliefs about curriculum, learning, and teaching" (p. 46). At later CERMEs similar expectations have been phrased differently, from using "practice" synonymously with "beliefs in action" (Furinghetti, Grevholm, & Krainer, 2001, p. 268), over talking about teachers' beliefs as "reflected in their classroom practices" (Ferreira, 2007, p. 1996) to claiming that beliefs "play a key role for [teachers'] decision-making" (Schueler, Roesken-Winter, Weißenrieder, Lambert, & Römer, 2015, p. 3255). Some of these wordings indicate that it is helpful to distinguish between espoused and enacted beliefs, and/or that contextual factors need to be considered to account for the role of teachers' beliefs in practice.

One of the challenges for belief research is that there is no agreement about an explicit definition of the key construct of beliefs. In spite of that, it has been argued that there is a core to how the concept is generally used (Skott, 2015). Beyond the expectation of impact there are three other characteristics to this core, namely that the mental constructs about which the term is used are subjectively true, value-laden, and relatively stable. Despite some agreement on this, core conceptual difficulties remain. This is apparent also in studies presented at CERMEs and is reflected in the multitude of terms that are used almost synonymously with beliefs, for instance conceptions, perceptions, views, worldviews, and priorities.

One particular aspect of the difficulties with the construct of beliefs is how to distinguish it from knowledge. A decade ago, Philipp (2007) suggested that one way to go about the quandary is to use the term belief about some conviction if one can accept an alternative or opposite position as reasonable and to call it knowledge if such alternatives are considered unreasonable. This suggestion does away with the absolutist connotations of knowledge, as the truth-value and justifications of knowledge now reside in social or individual domain. We have not come across similar wordings in CERME proceedings, but the intention of doing away with the absolute difference between knowledge and beliefs is sometimes apparent. An example is the knowledge quartet (KQ), where the

foundation dimension encompasses elements of both knowledge and beliefs (e.g. Rowland et al., 2013; Koklu & Aslan-Tutak, 2015; Samkova & Hosposova, 2015). Also, Kuntze et al. (2011) work with an overarching concept of teachers' professional knowledge that "integrates the spectrum between knowledge on the one hand and convictions/beliefs on the other" (p. 2620), because a clear-cut distinction between the two is unobtainable.

It is inherent in the notion of belief stability (cf. the core of the concept mentioned above) that beliefs take a long time to develop and that it takes substantial new experiences for them to change. Teachers' mathematics-related beliefs, then, are considered robust, and because they are an outcome of the teachers' own schooling, they are at odds with current reform efforts. This seems to be the dominant understanding also at CERMEs, irrespectively of whether the beliefs in question are about mathematics in general, about the teaching and learning of specific mathematical topics (algebra, data handling, number, etc.), or about particular aspects of the reform (e.g. problem solving or the use of ICT). It follows that belief change is considered important in many PD-programmes, and there seem to be three approaches to this. Sometimes the aim is to change teachers' beliefs by working with the history of mathematics (Figueras, 2003) or by engaging teachers in reflection on for instance their own mathematical activity (Hodgen, 2003), on their modes of classroom communication (Ferreira, 2007, 2011), on students' mathematical activity (Bas et al. 2013), or on the character of their own beliefs (Helmerich, 2013). Others base development initiatives on the expectation that although beliefs have some impact there is no linear and direct causality between beliefs and classroom practice. These studies draw on the distinction between espoused and enacted beliefs and try to make the latter (traditional) version align with the former (reformist) one (Climent & Carillo, 2001; Malara & Navarra, 2015). Still others adopt a comprehensive approach to teacher development and seek to change teachers' beliefs, knowledge, and practice in the same project, without a basing the initiative on a specific premise concerning the character of the connections among the three (Zehetmeier & Krainer, 2011).

It is generally acknowledged that beliefs are elusive and that there is no easy access to whatever teachers believe. The methodological problems in the field are also acknowledged in papers presented at CERMEs. In most of the studies, teachers' beliefs are inferred from – or attributed to – teachers on the basis of surveys, interviews, or observations from classrooms or seminars. A combination of these methods is sometimes used for purposes of triangulation (e.g. Rubio, 2011), or for other purposes (e.g. Guerreiro & Serrazina, 2011). A few studies directly address the methodological challenges of belief research and develop other approaches than the dominant ones mentioned above. For instance, Reid et al. (2015) ask teachers to select and discuss video recorded examples of typical and exceptional teaching from the teachers' own and from

their colleagues' classrooms. Positioning teachers as observers and analysing what they observe and what they say about their observations, Reid and his colleagues seek to gain insight "into the implicit criteria that guide [teachers'] observations." (p. 3116). It is indicative of the general methodological problems of belief research that in Reid et al.'s study the teachers focussed on other issues when asked to select typical and exceptional examples of teaching than when they discussed these examples with their colleagues. As Reid et al. observe, this points to a methodological problem, if one relies on only one type of data in studies of beliefs. However, the observation may also indicate that "implicit criteria" or beliefs are in some sense situated. If this is the case, the traditional argument for methodological triangulation may be challenged, as different methods do not necessarily provide access to the same mental constructs, beliefs.

We argued above that the expectation of belief impact was and still is a fundamental premise for research on teachers' beliefs. However, the thesis of congruity between the two has been refuted as much as confirmed in empirical studies (Fives & Buehl, 2012). Further, and as indicate above, belief research is almost notorious for its conceptual and methodological problems. Few studies in and beyond CERME react to these problems by challenging the key assumptions of belief research (see Larsen, Østergaard & Skott (2013) for an example). Over the last decade, however, one trend in the field of beliefs has been to rely less on expectations of linear and causal relationships between beliefs and practice and point to other types of connections between the two. This development may be seen at least in part also as a response to the conceptual and methodological problems of the traditional field of beliefs, and it is hinted at in the title of two volumes that at different points in time intended to present the state of the art in this field. The first one, edited by Leder, Pehkonen & Törner in 2002, is called *Beliefs – a hidden variable in mathematics education?* It seems that the variable they were looking for was at least to some extent considered an independent variable in relation to practice. The second book, edited by Pepin & Rösken-Winter in 2015, is entitled *From beliefs to dynamic affect systems in mathematics education*. This title suggests both that the relation between beliefs and practice could be viewed as less causal and more reciprocal than before and that dynamic interconnections among different mental constructs, including beliefs and knowledge, should be considered. This dynamic trend is also to some extent apparent in papers presented at CERMEs. Some focus on the belief-practice interface and point to the contextual framing of belief enactment (e.g. Turner, 2007). This challenges the expectation of belief impact. Others discuss beliefs as part of a broader set of teaching resources or competences (Bosse & Törner, 2015; Schueler et al., 2015; Wittmann, Schuler, & Levin, 2015) and by doing so question the isolated significance of the ones that are traditionally referred to as beliefs. In this sense developments at CERMEs reflect the more

general trend towards dynamic perspectives on the role of teachers in mathematics classrooms.

4. STUDIES FOCUSING ON PROFESSIONAL IDENTITY

Research on teachers' professional identities has gained momentum in mathematics education over the last decade as a supplement and to some extent as a challenge to studies of teachers' knowledge and beliefs. Generally, identity studies tend to adopt more participatory approaches to human functioning than studies in the two other fields. They draw, for instance, to a greater extent on social practice theory (e.g. Wenger, 1998) or discourse analysis (e.g. Gee, 2000-2001).

Identity has also become an issue in studies presented at CERMEs, but it still attracts less attention than research on teachers' knowledge and beliefs. Also, teachers' professional identity is often included in studies of the two other fields or of classroom practice, sometimes leaving the concept of identity and relationship between identity and other constructs somewhat underspecified. Identity, then, may be mentioned in passing and as an unproblematic reference to prospective or practising teachers who need to think differently about the profession and about themselves as teachers or teachers-to-be. This means that in many cases the notion of professional identity plays a relatively minor role.

There are, however, studies that explicitly link teachers' knowledge and their identity. At CERME 2, for instance, da Ponte and Oliveira (2001) report on an intervention study to support prospective teachers' use of ICT. They suggest that the participants' more profound understanding of the use of technology supports their development of a professional identity, that is, their "adoption of a standpoint and values of a mathematics teacher" (p. 311). In a later study, Nunes and da Ponte (2011) investigate how teachers interpret and manage formal curricula in collaboration with colleagues. They talk about the development of professional identities when teachers engage with "the culture, values and norms of the professional group, but also [have] the possibility of influencing and thus contributing to the change of the group, mobilizing his/her cultural background and personal experience." (p. 2788). As a final example, Sayers (2013) argues that elementary teachers' knowledge and use of the terminology and rhetoric promoted by official documents on school mathematics is a source of their professional identity and "the means by which they remain part of the primary teaching community" (p. 3250).

The notion of professional identity is often mentioned in connection with teachers' beliefs. This is hardly surprising, as one part of belief research is concerned with teachers' beliefs about themselves as learners, teachers, and doers of mathematics. However, it is also argued at times that teachers' beliefs about the subject itself may be a significant influence on their identity (Sayers, 2007). One relatively early study that linked beliefs and identity, but paid

significant attention to the construct of identity when doing so, is the one presented by Hodgen at CERME 3 (Hodgen, 2003). Hodgen defines and delineates the concept of identity by drawing on Wenger (1998) and Holland et al. (1998). He reports on a case study of a teacher, Alexandra, who through her engagement in a long-term PD-programme begins to view mathematics as sets of closely connected ideas and develops from viewing mathematical truth as established by others to seeing herself as participant in negotiating mathematical justifications. This development is, Hodgen suggests, an outcome of Alexandra taking on different identities during the programme, such as a teacher, a tutor, and a researcher. Moving in and out of these different identities requires and enables her to reflect on her own previous engagement with mathematics and develop her approach to the subject in the process.

Over the last few CERMEs other papers have also referred to e.g. Wenger and Holland et al. in studies of teacher identity. Bosse and Törner (2015) question the relevance of an exclusive emphasis on beliefs and knowledge in research on and with out-of-field teachers, that is, people who work as teachers of mathematics, but who do not have an educational background that formally qualifies them to do so. Bosse and Törner argue that identity, located somewhere on “a socio-cognitive continuum” (p. 2770), provides a useful perspective for understanding how these teachers engage with the practices of the profession. Building on Wenger’s notion of practice, they seem to locate their own take on identity towards the ‘social end’ of the continuum. Ebbelind (2013, 2015) refers to discourse analysis and a Patterns-of-Participation framework that builds upon Wenger (1998) and Holland et al. (1998) (among others) to analyse the development of prospective elementary teachers’ professional identities during their pre-service programme. He analyses how research participants’ past experiences and engagement with other current practices relate to their tales of themselves as prospective teachers.

While research on professional identity is still not nearly as prominent at CERMEs as studies of teachers’ knowledge and beliefs, it seems that the field is attracting increasing attention. This may reflect a shift in the overall paradigms used in the field from primarily acquisitionist understandings of what it is to learn, to know, and to believe towards a more participatory stance that focuses on teachers’ engagement in social practices as they unfold. This seems to be so at least to the extent that the former stance is to some extent supplemented with the latter. We return to this question in the concluding discussion.

5. UNDERSTANDING TEACHER- PRACTICE RELATIONSHIPS

While it is a continuous effort in the CERME community to understand relationships between or among teachers’ knowledge, beliefs, and identity on the one hand and the practices that evolve in their classrooms on the other, understandings of that relationship have changed over the last 20 years. As indicated previously in the case of beliefs, research on teachers and teaching has

increasingly moved towards a concern for the complex, dynamic and emerging character of classroom practice. This implies that a different set of frameworks may be useful, frameworks that acknowledge the significance of the multiple micro and macro factors that may influence how learning and lives in classrooms unfold.

Before embarking on our discussion of CERME studies focusing on mathematics classroom practices, a short note on the meaning of the term 'practice' and its development over time is needed. da Ponte and Chapman (2006) noted that the early uses of the term 'practice' in mathematics education research are mostly related to individual's 'actions', 'acts' or 'behaviours'. Gradually its meaning evolved to include what a teacher does, knows, believes and intends. Nowadays, two main considerations of the term can be identified, a cognitive and a socio-cultural. The former pays special attention to teachers' action plans and decisions taking into account their knowledge, beliefs and goals (e.g. Schoenfeld, 2000). The socio-cultural perspective, on the other hand, sees practice as a social phenomenon, that is, as "the recurrent activities and norms that develop in classrooms over time, in which teachers and students engage" (Boaler, 2003, p. 3). The use of the term 'practice' in ERME conferences appears to reflect the above evolution of its meaning.

There are shifting trends in the prevailing research interest in papers presented in CERME Working Groups devoted to teachers and teaching. In the limited number of studies appearing in the early CERMEs, the focus is on aspects of teachers' activity typically exhibited within mathematics classrooms. These studies focus for instance on classroom management of pupils' errors (Tzekaki, Kaldrimidou, & Sakonidis, 2001); on selecting and implementing tasks/examples (Rowland, Thwaites, & Huckstep, 2003); and on reacting to students' unexpected answers (Coulange, 2005). Such papers offer some insights into how the acts of teaching affect opportunities for students' mathematical learning, most often in elementary school classrooms. The frameworks used are often (explicitly or implicitly) constructivist/acquisitionist, and the methods are mainly qualitative and small scale, although they are sometimes combined quantitative approaches.

This line of research does not cease to appear at later CERMEs. Examples are the effort by Gunnarsdóttir & Pálsdóttir (2015) to shed light on how teachers structure everyday mathematics lessons or Drageset's (2015) study of teachers' response to students' answers, when the student does not explain how the result was obtained. However, two new research trends become evident. The first is similar to the one above, but concentrates on the study of 'desirable' instructional practices such as practices compatible with (reform) initiatives. The initiative may be concerned with problem solving (Asami-Johansson, 2011; Georget, 2007) or with integrating ICT in mathematics teaching (Abboud-Blanchard, Cazes, & Vandebrouck, 2007); with an effective teacher's actions in

the classroom (Taylan, 20125); or with the use of research findings (Nowińska, 2011). Overall, these studies employ similar theoretical and methodological approaches to the ones of the first era.

The second trend includes studies examining the dynamic nature of classroom interaction between teachers and students (Berg, 2011; Nunes & Ponte, 2011; Wester et al., 2015). This research recognizes the complexity of teaching and aspires to provide analytical tools to further conceptualise it (Badillo et al., 2013; Felmer et al., 2015). Sometimes one of three more specific foci is used. The first concerns communication in the mathematics classroom and emphasises teachers' orchestration of mathematical discourse, how one may deal with students' unpredictable contributions, mathematical argumentation, or effective questioning (Drageset, 2013; Kwon, 2015; Ponte & Quaresma, 2015). The second focus is on teaching and classroom dynamics in relation to more particular mathematical meaning such as algebra or statistical graphs (Kaldrimidou et al., 2011; Cusi & Malara, 2013; Velez & Ponte, 2015). Finally, the third focus is on teachers' reflection on their own teaching practice with emphasis on classroom interaction (Berg, 2011; Nunes & Ponte, 2011; Wester et al, 2015). Several different theoretical frameworks are used to address these issues, few of them cognitive and the majority interactionist or participatory in other ways. From a methodological point of view, all but few of the papers in this group follow an interpretative paradigm, conducting research through case-studies based on classroom observations of and interviews with practising or more rarely prospective teachers.

A third trend was already emerging in early CERMEs (e.g., Krainer, 2001), but has become more visible in the later ones. These studies do not only view instruction as habits or norms for actions facilitating pupils' learning, but also as situated/participatory activities that offer contexts for the teachers' own learning. The aim of this line of research is to explore how teachers' reflection on and inquiry into classroom practices contribute to their knowledge for teaching mathematics and to their sense of 'being' or 'becoming' teachers (Azcarate, Cardeñoso, & Serradó, 2005; Baş et al., 2008; Caseiro, da Ponte, & Monteiro, 2015; Ferreira, Martinho, & Menezes, 2013; Gade, 2015). This is so for example when teachers are involved in discussions of real classroom episodes (Clivaz, 2013) or seek to unpack students' ideas (Aslan-Tutak & Ertas, 2013). The relevant papers capitalize on multiple theoretical and analytical frameworks, but there is a trend to supplement or replace the traditionally dominant constructivist paradigm with more participatory ones. These include CHAT, social practice theory, or some form of interactionism.

6. CONCLUDING DISCUSSION

Research on teachers and teaching reported at CERMEs over the last 20 years is characterised by both change and continuity. One recurrent theme is how teachers contribute (or not) to the types classroom practice envisaged by current

reform efforts, and in particular how they deal with contingencies that may arise. Phrased in the terminology of the draft of the NCTM standards (cf. section 1), this amounts to if and how the teacher manages to supplement planned classroom activities with leading the class into unanticipated territory and – taking the metaphor a little further – getting the students home safe again. Another theme that has been dealt with continuously is what it takes for teachers in terms their professional qualifications to do so.

It is apparent from the previous sections that the latter question has often been addressed in studies of teachers' knowledge and beliefs. Sometimes knowledge and beliefs have been studied as characteristics of the individual teacher, and as if they function almost independently of one another. Also, it has often been a premise that they are the basis of teachers' acts and meaning-making and that each of them significantly impact practice. If taken to extremes, this means that "practice" is viewed as a function of two variables: teachers' knowledge and beliefs. For obvious reasons, this fuelled significant amounts of research on and with *teachers*, and, in line with the dominant theoretical framework at the time of the first CERMEs, teachers' learning as well as their knowledge and beliefs have often been interpreted in constructivist terms.

There is continued interest in understanding teachers' acts and meaning-making, including how they relate to the practices that unfold in the teachers' classrooms. However, the previous emphasis on teacher characteristics has been supplemented with a stronger focus on classroom practices. This shift of emphasis may be phrased as one from research on *teachers* to one on *teaching*. To some extent this challenges the idea that what teachers think and do in their classroom is determined by previously reified mental constructs, their knowledge and beliefs, and it suggests that more dynamic perspectives may be needed.

There are different versions of this more dynamic perspective. In some cases, the argument is that the multiple relationships among different teacher characteristics need to be considered together with the contingencies that arise as classroom interactions unfold. This approach may still be in line with the original acquisitionist orientation, but elaborate on it by working with relationships among more and more complex mental constructs as well as with the interface between these constructs and social setting in which teaching unfolds. Another more radical and less used interpretation of the dynamic or processual approach does away with the acquisitionist connotations of knowledge and beliefs. It draws on social practice theory, cultural historical activity theory, and sometimes on Sfard's commognitive framework, and its participatory stance is more in line with what Lerman (2000; 2006) calls the social turn in mathematics education research. Part of this trend includes a growing interest in teachers' professional identities, a term that is also interpreted in processual and participatory terms.

There are still many studies conducted on *teachers* in mathematics education, but the trend at present is moving towards studying *teaching* and adopting one of the two approaches to this outlined above. Both these approaches link the acts of teaching to the broader social practices of which they are part. One may wonder what the potentials are of each of them and whether the tensions between the frameworks should be seen as an opportunity for dynamic growth in the overall field of research on teaching or as an obstacle to the accumulation of knowledge – or possibly as both. The moral of the story is that, although there appears to be some consensus that this more dynamic and less causal approach is needed, this consensus is still relatively recent and it seems like there is little reason to opt for a forced choice between them.

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