# 13 MATHEMATICS TEACHER EDUCATION AND PROFESSIONAL DEVELOPMENT

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## Introduction

Research on mathematics teacher education and professional development has been a strong theme of CERME from its very beginning. It has focused on a wide range of issues connected with different activities of teachers, teacher educators and researchers, their influence on the participants' professional competence and/or teaching and impact of contextual variables connected with them. The practice of teacher development is complex, institutionally and culturally specific. Career structures for teachers affect their motivation to take part in professional development programmes (differently in different countries) and relationships between university and school need to be handled carefully. Small scale professional development work with schools tends to deal only with 'special teachers' whereas large scale work runs a risk of producing surface or naïve interpretations of 'reform' messages.

The chapter will identify and follow main ideas which were discussed during CERMEs and later published in proceedings. We shall attempt to compare and analyse how different theoretical approaches and research paradigms effect the results. We will consider an evolutionary approach, identifying the relevant themes that attracted the attention during several conferences (sometimes with different meaning).

Professional development is about becoming autonomous and critical at designing and conducting classroom teaching. We identified three strongly related stand points which were discussed during CERMEs:

1. Linking theory and practice in pre- and in-service teacher education
2. Balancing between autonomy of the teacher and cooperation with colleagues and/or researchers
3. Reflective practice

## The tension between theory and practice in teacher education and professional development

The tension between theory and practice was expressed by Seeger and Steinbring as the relation "between the teacher's immediate involvedness and critical distance" (Seeger and Steinbring, 1992 cited in Scherer and Steinbring, 2003: 1). In the process of teacher education and professional development of teachers we encounter to cope with the gap between theory and practice constantly. When one deals with theory and practice, the former is frequently associated to research or academic context (the content to be learnt in the pre-service institutions), and the second is associated to teachers (e.g. Schön (1983, p. 26) distinguishes amongst "practitioners, educators and researchers"). Amongst the conclusions of a European project for professional development, Malara and Tortora (2009) identified, as an area of teachers' weakness,

"the bifurcation between theory and practice (e.g. difficulties in realising what has been studied or planned)" (p. 1804).

From the very beginning of CERMEs, the connection between theory and practice was in the spotlight. One of the main goals at CEME1 was "Focusing on research…in order to find bridges between theory and practice in teacher education" (Krainer, 1999, p. 23). At CERME2 the investigation on the relationship between theory and practice emerged as the first philosophical goal for the future of the working group. Apart from that, one of the special interests groups formed at the conference was about this relationship, its aim being "to study how theories may affect the practical knowledge of teachers and how teachers themselves perceive that their practice fits with theoretical issues of mathematics education" (Furinghetti, Grevholm and Krainer, 2002, p. 268). A schema was presented for the discussion, in which elements of knowledge, theories, experience, context, beliefs and practice were represented. The relation between theory and practice became the title of one of the working groups at CERME3 and the title of a plenary panel conducted by Barbara Jaworski at CERME3, showing the importance of this issue at that time. The relationship between theory and practice, understood as between university and schools, was one of the emerging issues at CERME5: "How can we help pre-service teachers to bridge between theory and practice, or between the university and school context?" (Carrillo, Santos, Bills and Marchive, 2007, p. 1825).

Several fields or issues come across when focusing on the mentioned connection. Carrillo, Coriat and Oliveira (1999) matched this connection to several fields such as, for example, "an equilibrated conception of mathematical knowledge [and] the school practices for prospective mathematics teachers" (p. 103). Sometimes, even, one can speak of several theories and several practices. Jaworski (2004) pointed out several perspectives on theory and practice, moving the attention to the fact that teachers' theories exist and differ from those of teacher educators and researchers. In the discussions of the group at CERME3, theory was, for some participants, related to the work of the researchers, and practice to that of teachers at the schools (Jaworski, Serrazina, Peter Koop and Krainer, 2004). In the introduction of the WG 10 at CERME 6 related poles were mentioned: "Several themes are possible to be discussed, such as […] links between theory and practice, research and teaching, and teacher education and collaborative research" (Santos, Carrillo, Hošpesová and Abboud-Blanchard, 2009).

Østergaard (2015) adopts a multifaceted panorama of the relationships between theory and practice: "The ATD [Chevallard, 2012] model points out three different theory-practice problems in mathematics teacher education. It is crucial to put focus on all three axes and give student teachers opportunities to establish coherence between theory and practice in connection to the three axes" (p. 2894). In the proposed teacher education model, one axis "stresses the divide between practical, procedural mathematics with emphasis on techniques to carry out tasks and theoretically doing mathematics by combining techniques and concepts, arguing, proving etc." (p. 2890). The other two "theory-practice axes are dividing, respectively, the scholar mathematics

and knowledge meant to be taught and knowledge meant to be taught and knowledge actually taught" (p. 2890).

The role of theory in the work with teachers is a major concern, as sometimes researchers intend to apply their theories to practice (e.g. asking teachers to implement some theoretical results or methods), and other times they try to get insight from practice to develop their theories, a crucial question remaining to be tackled: what kinds of relationships between theory and practice are beneficial for both researchers and teachers? And, also important, what roles should/can/do teachers and researchers (like to) play in the domain of this relationship? From the point of view of other participants, if we conceptualize the work at the universities (as teacher educators) as something more than creating theories, and the work at the schools as more than applying theories, then the relation between theory and practice becomes smoother.

The transition from pre- to in-service education is approached several times. Carrillo et al. (1999) say that "the subjects are usually theoretically biased and one expects that the prospective teachers shall make the integration…when they become teachers. And this perspective goes on in the in-service education" (p. 135). In the study of Verhoef and Terlouw (2007), questions deal directly with the relationship between trainee teachers' experiences and "the teachers' role in the classroom setting with reference to discussed theory and trainers' didactical remarks" (p. 2017). Cusi and Malara (2011), being aware of the lack of relationship between theory and practice amongst undergraduates who have not yet worked at the schools, teach a course whose main goal is "giving them theoretical and methodological tools to learn how to interpret their future actions in the classes" (p. 2625).

Collaborative environments (approached in another section in this chapter), in which researchers, teachers and student teachers work together, reflecting on and designing classroom situations, emerged as an appropriated context to build a bridge between theory and practice. García, Sánchez, Escudero and Llinares (2004) envisage this relation under the notion of community of practice (Lave and Wenger, 1991) (of teaching), where their student teachers are involved. The construction of a professional language by student teachers in a learning community using lesson study is seen by Gunnarsdñttir and Pálsdñttir (2011) as an example of connection between theory and practice.

Mgombelo and Buteau's (2009) in their paper, they present a collaborative work whose goal is to connect research and practice in the setting of initial mathematics teacher education. Therefore, here the envisaged relationship takes place not between, for instance, university and schools (as stated before), but within university. Mgombelo and Jaipal-Jamani (2011) propose collaborative environments (teachers and researchers) as contexts where teachers "can experiment with new ideas in theory and practice" (p. 2774).

The substantial learning environments (Wittmann, 1995) are considered by Tichá and Hospesová (2011) as "one of the fields where researchers' and teachers' objectives

interlink, where not only theory and practice, but also mathematics and didactics of mathematics mingle and which is open to natural long-term, systematic cooperation of researchers and teachers" (p. 2890). In this sentence, Tichá and Hospesová (2011) include the cooperation between researchers and teachers as a central aspect in the link between theory and practice. In a context of development programmes, Kuzle and Biehler (2015) analyse mathematics educators' practices and say that "an intertwinement [amongst mathematics educators and teachers] of input, active learning and reflection phases…is crucial for connection between theory and practice" (p. 2849). Theory and practice are also connected in the project presented by Martignone (2015), in which teachers and researchers "analysed together the laboratory sessions: in particular focusing on the exploration processes, the conjectures productions, and the proof constructions in the activities with mathematical machines" (p. 2870).

The analysis of classroom episodes is seen as a means to link theory and practice. Concerning pre-service education Goffree and Oonk (1999) contributed a way for student teachers to link theory and practice through the analysis of experienced teachers' practice. Krainer (2003) presented investigation into their own teaching at the core of the relationships between theory and practice for teachers and educators (both groups being considered as practitioners). Another perspective on the mentioned relation came from Brown's (2006) paper. Adopting an enactivist position (Reid, 1996), she presented how she dealt with her own practice as teacher educator, in which theory is the result of a progressive awareness of what happens in practice. The analysis of classroom episodes in pre-and in-service settings, differentiating actions and cognitions (including beliefs and knowledge), is presented by Ribeiro, Monteiro and Carrillo (2009) as a good "starting point for an approximation between theory and practice. It would mean that researchers and teachers 'speak the same language', using the same codifications; in doing so, a great degree of collaboration is needed" (p. 2037). The authors underline the role of collaboration in the process of linking theory and practice. Berg (2011) approached the link between theory and practice within the context of a developmental research and using the notion of inquiry cycle (Jaworski, 2007).

Other notions or models have been applied to explain processes related to the link between theory and practice. Skott (2006) introduced the notion of theoretical loop to explain the process of starting in teachers' practice, go on theorising it and coming back to practice. This loop pays special attention to the usefulness of theory in practice: "It is reasonable to assume that theoretical constructs grounded in the sites of practice are of greater potential use to practitioners (e.g. teachers) than constructs developed without such grounding and without recognition of the contextual complexities of teaching" (p. 1599-1560). García, Sánchez and Toscano (2015) implement a dynamic model relating theory and practice, in which they start focusing on the needs coming from practice (pre-service teachers' future classrooms), continue reflecting on theoretical elements that could face the previous needs, come back to practice through

the design of tasks, and restart again. This model is applied to the design of tasks in primary mathematics teacher education.

Of special relevance is the association between professional development and the relationship between theory and practice. The interest of teacher educators in promoting (student) teachers' investigations and in researching their professional growth is seen by Krainer (2002) as essential aspects "for improving theory and practice of teacher education" (p. 289), and follows: "this investigative attitude of the teacher educators might contribute to (student) teachers' views that investigations are really decisive for promoting professional growth" (p. 289), linking this way theory (investigations) and practice (teachers' investigations and growth). In Carrillo, Climent and Muñoz (2006), the case under study serves as a means to reflect on the authors' former conceptualisation of professional development. Thus, a process of, first, conceptualisation, second, the practice of a novice teacher and, third, reconceptualization of professional development, shows a singular relationship between theory and practice, where the analysis of the latter introduces new considerations in the theory previously built. An attitude of teachers toward theorising their practice and question theories is considered by Zehetmeier (2009) as one of the factors that promote the impact of professional development projects. In this case, a mode of relationship between theory and practice serves as a means to develop professionally.

A different way of thinking of the relationship between theory and practice emerged at CERME4. In the introduction of WG 12 Carrillo, Even, Rowland and Serrazina, 2006) summarised the emerging issues at the conference: "One of the issues that arose in the group was the demand for theories, perspectives and methods to contribute to the way of thinking about the classroom. It means that they should try to capture or approach the flavour and essence of the classroom activity" (p. 1407). Therefore, in this case, what is underlined is the need to capture the essence of classroom activity, and this entails a theoretical concern as well as a coherent methodological approach.

The pre- and in-service courses are, in some investigations, analysed or used from the perspective of the relationship between theory and practice. Malara and Navarra (2007) approached the promotion of this relationship through the implementation of early algebra tasks in in-service courses, the aim being "to test the effectiveness of such task as an element of mediation between theory and practice and, more specifically, as a tool to set trainees 'inside' a virtual class, so that they can get aware of what it means to 'act in the moment'" (p. 1930).

Joubert, Back, De Geest, Hirst and Sutherland (2009) present an in-service course driven by an advisor and a teacher educator where the participants were asked to read some research papers and reflect on their application on their own teaching contexts. The role of the course leaders (and we would say that also the convenience of the relationship between in-service and pre-service training institutions) is emphasised by one of the participant teachers: "they [the course leaders] provide a link between the

theory and practice in both [my] own classroom and what other schools are doing" (p. 1767).
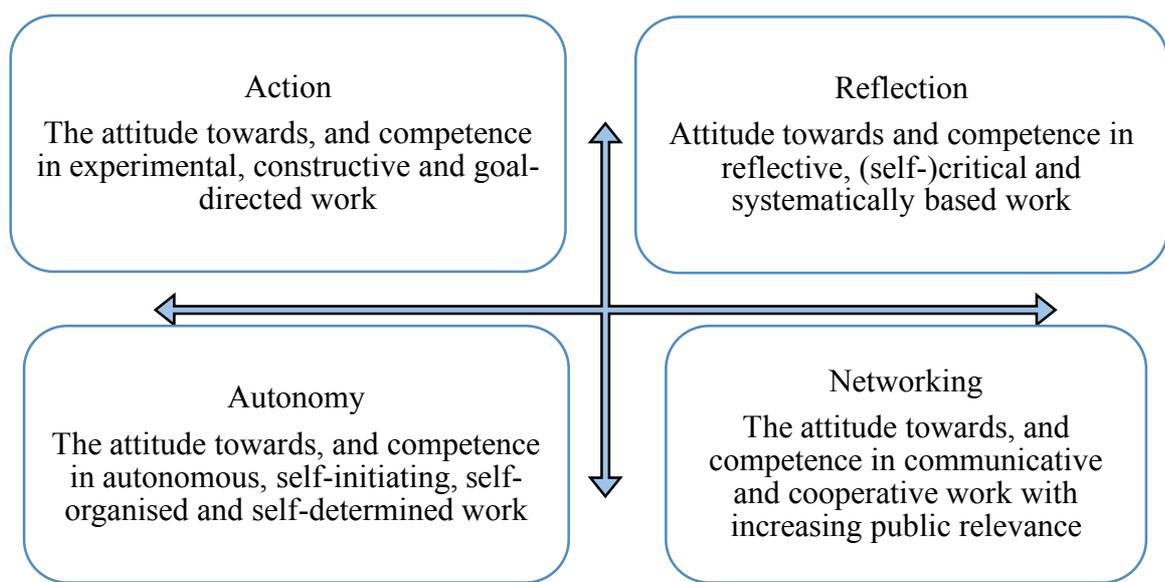
Aizikovitsh-Udi, Clarke and Star (2013) contribute the idea that connecting theories and practice in in-service programs is essential, but not enough to provoke teachers' change in the classroom. Berg (2013), in a course for novice in-service teachers, uses the notion of inquiry cycle, "as a means to engage in life-long learning while recognising the importance of results emerging from research in mathematics education" (p. 2947). The lack of influence of methods course on teachers is commented by Somayajulu (2013): "One likely reason for this is that the methods courses were designed without much sensitivity towards helping the teachers' bridge theory and practice. In light of the findings of this study, I recommend that pre-service teacher education programs should take into consideration the prior experiences of teachers as learners and build on them while designing activities that are more effective" (p. 3275).

Finally, coming from the investigations into teacher knowledge, Ribeiro and Carrillo (2011) propose moving beyond the analysis of a teacher's knowledge to contribute to the improvement of his/her practice, that way contributing to build a "bridge between theory and practice" (p. 2824), helpful in initial as well as in in-service education. Ribeiro, Aslan-Tutak, Charalambous and Meinke (2015) write that focusing on what teachers know and how they know it can "lead to the development of practices that enrich the levels of awareness and connections that contribute to improved education and, ultimately, practice. Bridging theory and practice is essential for such improvement, and core to building such bridges is defining the nature and goals of proposed tasks…for enhancing teacher acquisition of ideal knowledge that would allow them to foster fruitful mathematical understanding in their students" (p. 3179).

## Collaborative environments in teacher education and professional development

Since the very beginning of CERME it is possible to find some references concerning the importance of collaboration in teacher education and professional development. In particular, mathematics teachers engaging in research into their own practice are considered as opportunities to their professional development: "Collaborative work between teacher researchers, teacher educators, and academic researchers. Perhaps such collaboration can be extremely fruitful in the enhancement of mathematics teaching and development of related knowledge in the public domain" (Jaworski, 1999, p. 213). Later Krainer presented for the first time at CERME theoretical framework that highlight cooperative work as one of the component of the networking dimension included in a four dimension-model of teachers' professional practice (see Fig. 1).

According to the author: "The (student) teachers share their experiences with other participants or with the teacher educators, they use electronic means and research literature, and thus enriched their personal views (and those of others)" (Krainer, 2002, p. 289).

| Action | Reflection |
|---|---|
| The attitude towards, and competence in experimental, constructive and goal-directed work | Attitude towards and competence in reflective, (self-)critical and systematically based work |
| Autonomy | Networking |
| The attitude towards, and competence in autonomous, self-initiating, self-organised and self-determined work | The attitude towards, and competence in communicative and cooperative work with increasing public relevance |

**Fig. 1 Four dimensions of teacher´s professional practice (schematized in accordance with Krainer 1996).**

So, collaborative work may be constituted by several groups of persons. In the case of a group of teachers, Witterholt and Goedhart (2009) studied the development of practical knowledge in Statistics of two teachers working together during a period of time. Although these teachers maintains their different ideas face to what is to teach, they were committed with the success of the peer work and "experienced the interest of combining each other's ideas and constructing an educational design to which everybody could commit (p. 1998).

Nunes and Ponte (2009) presented a study that aim to understand the curriculum management developed by a mathematics teacher and the relationship between such curriculum management and the collaborative work undertaken by the mathematics teachers' subject group. The results pointed out that the curriculum management contains a collective level (the annual planning and the construction of units and tasks) and an individual one (decision about what is important to develop with their students, such as the autonomy in mathematics learning, the responsibility in their own actions and the capacity to think independently). Some conflicts appear into the group when different perspectives were in confront, but the mathematics teacher participating in the study, "the natural leader of the group, nurtures his relationship with his colleagues using curriculum management as a focal activity" (p. 1721). Moreover, the dynamics from the collaborative context "support the professional development of the teachers and their capacity to accept new challenges" (Nunes & Ponte, 2011, p. 2795).

Other context of collaborative work includes teachers and researchers, in in-service or pre-service educational environment. These studies were the ones that appeared along the CERME more often, being in alignment with the assumption that "the cooperation of teachers and researchers in mathematics education represents a broad and relevant topic" (Novotná, 2015, p. 62).

If to be able to work collaboratively is an important capacity for teachers, it is expected that pre-service teachers education programs pay attention and creates favourable contexts to develop such capacity. That context may be create trough certain type of activities such as the development of project work, which requires "predisposition and the teacher's ability for self-exposure" (Santos & Bento, 2007, p. 1974) or planning and discussion collaboratively with the teacher educator, following a design of lesson study (Gunnarsdñttir & Pálsdñttir, 2011). Whatever is the focus of the collaborative work, the problem of sustainability of the learning that occur in this context in pre-service environment is an important issue (Gunnarsdñttir and Pálsdñttir, 2013). According to these authors, in the perspective of the newly graduated teachers, the collaborative practices they experience in their pre-service teacher program were important not only during the program but also in their practice as teachers. Nevertheless, they still felt the need of support from outside the school.

In in-service educational environment, the studies focused for example, in planning teacher activities (Martins & Santos, 2011), namely in task design using an experienced based approach to identify the need of supporting theories (Nilsson, Sollervall, & Milrad, 2009), in the process of interaction between educational professionals (Muñoz-Catalán, Carrillo, & Climent, 2009), in the teacher discourse in mathematics classroom (Martinho & Ponte, 2009), or to support changes in assessment practices (Dias & Santos, 2015). In all of the studies, the collaborative context has been considered an important support to the development of teachers' professional knowledge, even if their development must be continued.

We have been presenting results from different studies that considered a collaboration setting. But until now the meaning of collaboration was not clarified. We can remark reading the different CERME proceedings that it is very rare that the authors explicit which is the meaning that they are considering for collaboration. There emerges a common idea that collaboration implies to work jointly, but for several authors, "Collaborating is not just sitting or working together" (Santos, Carrillo, Hošpesová, & Blanchard, 2009, p. 1680)". Once a while, we may find a clarification of the term collaboration. For example, Pesci (2009), focusing on the content of the collaboration process, wrote: "the term collaboration has a more general meaning: a positive inter-relationship amongst the people involved, not necessarily connected to a specific modality of acting in groups" (p. 1987). Berg (2011), highlighting the type of group' constitution, defined collaboration as "Teachers and researchers work together as co-learners" (p. 2588). Looking to the proceedings in a sequential order of publication we may remark that others contexts are progressively included instead of the collaboration contexts. This is the case of Wenger's (1998) community of practice (for ex. in Reinup, 2009) or Javorsky' (2008) inquiry community (for ex. in Oliveira & Henriques, 2015). When these new concepts were introduced, they were defined and clarified, instead to what we observed concerning the term collaboration.

Nevertheless, there are differences in this collaborative context between the members of the group. According to Bräuning and Nührenbörger (2009), teachers react "more

spontaneous than open – reflected (paraphrase, interpret) and use mainly knowledge by observation and experience and rarely knowledge by transfer and interrelation" (p. 1744). Moreover, although teachers and researchers seem, in general, to have identical objectives, that is not always really true. "Researchers look for answers to theoretical questions, while teachers deal with practical problems" (Hošpesová, Tichá, & Macháčková, 2005, p. 1914).

**Approaching reflection in mathematics teachers' education and professional development**

Reflection was discussed as an important ingredient in the process of professional enhancement. As starting point of the discussion on reflection and its role in teachers´ professional lives can be taken Krainer´s (1996) idea about the non-balanced relationship among four dimensions of the teacher's professional practice (depicted schematically in Fig. 1). He stated: "… there is a lot of action and autonomy but less reflection and networking ..." (Krainer, 1996, p. 310). In the paper several reasons for teachers' lack of reflection and networking are named: (a) the cultural and historical background; (b) no place for reflecting or networking in teachers' pre-service or in-service education; and (c) "reflection on one's own teaching and sharing it with colleagues is unusual, and costs a lot of time and effort." (p. 312).

Concept of reflection was among the discussed topics as a promising way in pre-service and in-service teachers' development. Climent & Carrillo (2001, pp. 270-271) said that "... the most important thing is to make available opportunities for reflection on one's own knowledge and beliefs, this being a possible point of departure for change chosen by the teacher". Jaworski (2003) suggested to investigate reflexive pairs, one of which is "inquiry and reflection". Scherer & Steinbring (2003, p. 2) claimed that it is necessary to create opportunities for "common reflection on everyday instructional activities".

Although the concept of reflection has different meaning in different papers, it mostly includes (a) pedagogical considerations as observation and contemplation about taken decisions, and used strategies; (b) often mathematical (didactical) analysis of key content elements, possible ways of explanation. Hodgen stressed the relation to teacher´s practice and spoke about "reflection as the reconstruction of experience and knowledge" (Hodgen, 2003, p. 1). Other authors agree that the reflection creates space for the transition from an intuitive to a conscious and justified action. "It is possible to treat reflection connected with interpretation of teaching/learning situations as the best way to develop the teachers' professional way of thinking and to present practical didactical theory." (Slavik, 2004, p. 1 our translation). Hošpesová and Tichá (2005) emphasised the analysis of the teacher's own thinking and dealing with pupils in a way suitable for their ability to plan their own lifelong education and consider qualified pedagogical (self-) reflection as one of the teachers´ competence.

Some authors stressed a necessity to systematically carry out and develop not only self-reflection but collective reflection, too. Scherer and Steinbring (2003) emphasised that

a mathematics teacher's work is so demanding that it is necessary to come to a qualified conception of collective reflection of everyday teaching activities to enable teachers a deeper view of their activity to help them.

Most frequent issue discussed during CERMEs were the changes observed in the research situated within the co-learning inquiry paradigm (Jaworski, 2004). It was shown that being involved in action and reflection collaboratively enables the participants to achieve a deeper understanding of both their own world and the world of the other participants in the community (Pesci 2007, Berg 2007). Georget (2007) summed that even simple opportunity to discuss or to have a reaction on own teaching helps the teachers to access existing documents and integrate them in their own practice. It was stated (Hodgen, 2003: 8) that to replicate such intense experiences for teachers "would be an extremely difficult and costly task" and he suggested a model of teacher education in which "teachers not only engage critically with the mathematics curriculum as teachers and as learners of mathematics, but also places them in situations where, as teacher tutors and curriculum makers, they encourage other teachers to engage critically in similar ways."

Several authors connected collaboration with reflection. Analysing classroom practices trough video episodes a collaborative discussion and a joint reflection were undertaken (Liston & Gill, 2011). This collaborative context included primary and lower secondary school teachers, student teachers, doctoral students in didactics of mathematics, teacher educators, and researchers. In this scenario, "the collaborative discussions support teachers' attempts to unpack the practice observed on the screen, through the implementation of mechanisms such as ascribing goals and weighing alternatives" (Karsenty, Arcavi, & Nurickp, 2015, p. 2831). Moreover, it is possible to teachers to get external points of view, contributing for a better understanding of their practices (Muñoz-Catalán, Carrillo, & Climent, 2007; Ponte, Serrazina, & Fonseca, 2003), "to achieve a deeper understanding of both their own world and the world of the other" (Berg, 2007, p. 1839) and to pay more attention to the importance of the role of error in mathematics learning (Guerreiro & Serrazina, 2009).

As other modes of intervention specific tasks (Pesci 2009, Helmerich 2013), online environment (Sánchez 2011), portfolios (Azcárate, Cardeñoso, Serradó, 2005; Santos, 2005), the role-play experience (Lajoie & Maheux, 2013) were discussed and presented.

It is necessary to state in agreement with the summary of TSG 18 on CERME 9 (Zehetmeier, Bosse, Brown, Hošpesová, Malara, Rösken-Winter, 2015) that the issues of (self)reflection are connected with other issues in teacher education, for example teachers' practices, content and methods of professional development courses, competency of didactical analysis, pedagogical content knowledge, didactic transposition, anthropological theory of didactics, content representation, spatial visualisation ability, teaching experiments, common games and childhood education.

**Emerging issues**

Research of mathematics teacher education and teachers´ professional development has been a stable focus over recent CERMEs. Research activities were broad and have varied from country to country due to different models of education and teacher education particularly. As mentioned by Zehetmeier et al. (2015, p. 2730) "research has increasingly focused not only on the participating teachers, but also on the role of teacher educators and academic researchers. So far, the research community has attempted to develop theoretical and methodological frameworks to both describe and explain the complex topic of mathematics teacher education … ".

Research community formulated several issues to be dealt in the future (Zehetmeier et al., 2015, pp. 2731-32), especially:
How do we deal with different perspectives of groups of participants in teachers' education (mentors, teachers, prospective teachers, teacher educators)?
How can we assess effects (immediate and long term) of supportive programs for (prospective) teachers or teacher educators? How to cope with low sustainability of such programes?
How can we actually observe identity development in terms of practice?


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