# 18 THEORETICAL PERSPECTIVES AND APPROACHES IN MATHEMATICS EDUCATION RESEARCH

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## 1. INTRODUCTION

Dealing with different theoretical perspectives and approaches in mathematics education is in accord with the spirit of ERME, especially with regard to communication, cooperation and collaboration. At the time of writing this chapter, six working groups (WG) on theoretical perspectives have been organized. The first was held at CERME 4 in 2005. This chapter offers a reflection on the development of the WG since then. Besides the WG, two plenary sessions have been devoted to the theme of theories, one at CERME 5 in 2007 and one at CERME 6 in 2009. We are aware that the perspectives about research and the place of theory in research, as well as the metalanguage which accompanies these perspectives, are not neutral. Nevertheless, we will use the terms "theory", "theoretical approach", "perspective" and "theoretical framework" interchangeably.

The chapter is structured as follows: Section 2 discusses the discourse on theories in mathematics education research (MER) until 2005. This short overview serves as background for the next sections. Sections 3 and 4 include sustained development of ideas by the CERME working group on theoretical perspectives over a number of CERME conferences. They are the core of the chapter. They are presented as separate sections but the issues are closely interrelated. Section 3 deals with the diversity of theoretical perspectives and networking theories, understanding how theories can be successfully connected while respecting their underlying conceptual and methodological assumptions. In section 4, questions concerning the role of theory, the relationships between theory and practice and the dynamic character of theory are discussed with regard to the diversity of theoretical approaches and with networking in the background. Section 5 deals essentially with ideas that arose within CERME and initiated or influenced, at least partially, developments in mathematics education beyond ERME. In

the last section, we reflect on what has been achieved within the working group and outline a vision for the future.

## 2. THEORETICAL PERSPECTIVES CIRCA 2005

The CERME WG on theoretical perspectives was formed in 2005 but it was not formed in a vacuum. The purpose of this section is to outline the 'theoretical context' the WG was born into. Due to word length constraints we do not explain the theoretical perspectives to which we refer but expositions of these frameworks can be found in *The Encyclopedia of Mathematics Education* (Lerman, 2014).

Cognitive perspectives dominated MER in the 1980s and the influence of Piaget's developmental psychology was strong. In Anglophone MER this influence can be seen in radical constructivism (Von Glasersfeld, 1995); in Francophone MER this influence can be seen in the Theory of Didactic Situations (Brousseau, 1997). But this influence was not uniform. For instance, as Radford (2008, p.320) points out, the dialectic between cognition and the social realm differ:

> For constructivists … the social realm is considered as a mere facilitator of the individual's development of these structures. In the Theory of Didactic Situations, cognition is also conceived of in an adaptive manner, but the social realm is thematized as a "milieu" and a game that the individual plays with it.

By the end of the millennium many researchers were influenced by 'the social turn', "the emergence into the mathematical education community of theories that see meaning, thinking, and reasoning as products of social activity" Lerman (2000, p.23). The social turn represented a wide variety of theoretical perspectives (activity theory, the anthropological theory of the didactic, situated cognition, social constructivism and more). The period around the end of the millennium also witnessed increased communication between mathematics education researchers across (and beyond) Europe, coupled with mutual attempts to understand the others' theoretical perspective (which we call 'networking theoretical perspectives'). An example of this is the forum in 2002 of *The International Group for the Psychology of Mathematics Education* (PME) devoted to mathematical abstraction in which representatives of different theoretical perspectives were invited to engage in critical dialogue.

Turning our attention to CERME we note that theoretical perspectives were explicitly considered before 2005. One example is in the WG on *Research Paradigms and Methodologies and their relationship to questions in mathematics education* (Hejny et al., 1999) at CERME 1. The focus of the group was "the nature, roles and function of theories in mathematics education" (p.212) and questions discussed were: What is a theory and how do theories arise? What is the role and function of theory in the research process? Is it feasible and useful to distinguish between different kinds of theories? How can theories in mathematics education be evaluated? This WG also noted that the research paradigms discussed seemed to be bound to the cultural or national contexts in which they were developed.

Theoretical perspectives were also raised in CERME discussions on empirical issues in MER, for example, in the WG *Inter-Relating Theory and Practice in Mathematics Teacher Education* (Jaworski et al., 2003) at CERME 3. This WG focused on the relationship between theories and practice and questions discussed included: what is the differences between applying theory and practicing theory; does a theoretical framework allow us to make more sense of what we see from reflection on and analysis of practice; to what extent should theories be explicit or implicit for researchers and teachers; and is practice a theory generator and does theory impact on existing practice?

So we see that, by the time of CERME 4, networking theoretical perspectives was an emergent practice amongst some mathematics education researchers.


## 3. PLURALITY OF THEORETICAL PERSPECTIVES: NETWORKING THEORIES

In this section, we analyze the progression of knowledge through the work of the WG during the different CERMEs. The following subsections are presented as questions that motivated the work of the WG. We describe attempts to address these questions as well as positive outcomes from these attempts.


**Diversity of theories**

The discussion in the WG started with the questions:

*Why diversity? Is diversity a necessity in the field or the sign that the field is in an immature state?*

At CERME 4 Artigue et al. (2006) observed that the plurality of theories results from the influence of theoretical constructions and approaches initially developed outside the field of MER (psychology, sociology, anthropology, mathematics, linguistics and epistemology) and that progressively become genuine constructions of mathematics education. They also observed "a more intrinsic diversity linked to the diversity of educational cultures, and to the diversity of the institutional characteristics of the development of the field of mathematics education in different countries" (*ibid*, p. 1240). In her plenary lecture at CERME 6, Bikner-Ahsbahs (2010) argued that the diversity of theories is scientifically necessary. Already at CERME 4, the WG participants agreed that diversity is both a source of richness and a source of fragility for research if we do not make scientific efforts to counterbalance the difficulties that stem from communication. These considerations led to new questions:

*How to deal with diversity? How to interact and to know each other?*

Plurality appears as something that simultaneously hinders and advances communication and dissemination and is an important question to deal with. The question of the multiplicity of theoretical frameworks and its source of richness appeared several times in the WG in the different CERMEs. Diversity is considered as a challenge for a community which intends to enable communication and progress between researchers of different theoretical frameworks. Therefore the necessity of learning how to deal with the diversity, complexity and richness of theoretical perspectives in mathematics education has been continuously raised in the WG. It is interesting to note that there was a general agreement that convergence in research would be beneficial in discussions at the first WG.

**The idea of networking**

Raising the question of dealing with the diversity of approaches was a first step. The expression 'networking' soon appeared in the WG as a way to highlight the question

without directly answering it. It includes another aspect of diversity, ways of making theories interact. Important questions arose from various WG discussions:

*Why is networking important? What are the aims of connecting theories? What are the problems which arise in the efforts to network theories? To what extent does the networking depend on the type of theories that are considered?*

There was a consensus in the WG on the importance of networking as a first step on the "path towards establishing mathematics education as a scientific discipline" (Artigue et al., 2006, p. 1242). The benefits and problems which arise in the efforts to network theories are described in the following subsections.

## Explicitness and awareness

*To what extent do we share the same notion of theory? Do we share the same degree of explicitness and awareness of the theoretical assumptions?*

Explicitness and awareness of the underlying assumptions of each theory serves as a basis towards networking.:

> It would be useful to make explicit the level at which a theory operates… Only on the basis of such awareness (of the theoretical assumptions), can a discussion on the possible coherence of underlying assumptions begin to take place so that a common language supporting such networking can be developed. (Artigue et al., 2006, p. 1243).

Among the underlying assumptions of each theory, "the assumptions on the nature of mathematical knowledge appear to be one key point for the analysis of similarities and differences between approaches" (Arzarello et al., 2007, p. 1622). The nature of knowledge and learning as individually or socially constituted was discussed in the first WG as a difference that might be an obstacle in connecting theories. But by the second WG cases of networking were analyzed that demonstrate attempts of new conceptualizations of the individual/ social interplay. In these cases, networking appears as a way to reflect on one's own work from the perspective of others. It enables researchers to be aware of their own assumptions and insights offered by other theories.

## Aims and limitations of networking

*What does networking mean? Is networking possible? What are the different aims of networking? What are the characteristics of the different cases of networking theories?*

Even at the outset, in the CERME 5 WG, comparing different initial strategies for networking made it clear that networking was important. Different aims in the efforts to network theories were differentiated. In some cases, the researchers were interested in the complementary insights that are offered when given data or an empirical phenomenon is analyzed with different theories. In other cases, the interest in the rich diversity of theories was to explore the insights offered by one theory to the other. In Arzarello et al. (2007, pp. 1625-1626) examples of different profiles of networking are presented; it also includes an initial attempt to understand the use of different theoretical perspectives in research on technology enhanced learning in MER. The next step was to explore the limits of such networking efforts and the difficulties encountered in connecting theories (Kidron et al., 2007)

**Different kinds of dialogues/interactions between theories**

*Is it possible to use more than one theoretical framework? What do researchers do when they use more than one theory? Do different approaches use the same words with the same meanings?*

In analyzing the source of problems which arise in efforts to network theories, new discussions took place in the WG at CERME 6. The participants distinguished between different kinds of dialogues between theories. The word 'dialogue' was used because networking does not necessarily imply changes in the theories at stake. The word 'dialogue' was also used not only to describe that which enables mutual understanding in the way we communicate our theories but also to emphasize differences in the use of language. The WG observed that even 'neighbouring approaches' (theoretical approaches which were born in the same educational and didactic culture) do not use the same words with the same meanings. This fact was observed in the different kinds of dialogues between neighbouring approaches that are offered in the papers by Ligozat and Schubauer-Leoni (2010), by Sensevy (2010) and by Artigue et al. (2010). These dialogues were explored in the WG and important differences between theories were observed.

**Towards a theory of networking theories**

*What is the methodology for 'networking theories'?*

The idea of a "theory of networking theories" emerged at CERME 5 ( Arzarello et al., 2007, p. 1627). The challenging task of networking 'approaches for networking theories' was an important part of the discussion in the WG at CERME 7. A central term that emerged from the WG at CERME 7 was 'transformation'. After networking researchers are often transformed and are able to see things which they could not see before. Conditions which permit fruitful dialogues were discussed in the WG. Artigue et al. (2011) consider the potential offered by the Anthropological Theory of the Didactic (ATD), with its central notion of praxeology, for addressing issues of networking between theories. Other approaches for networking were proposed in the WG and an effort of networking "approaches for networking theories" started (Kidron et al., 2011, p. 2377). It was also noted that strategies for networking depend to an important extent on how close (or far) the networked theories are located in the semiosphere (Radford, 2008). At CERME 8, the work continued with a new focus on theory and methodology. Questions were raised about the place of methodology in theoretical frameworks and the relation of this issue to the methodology for networking theories as described in Radford's semiosphere (2008) and Artigue et al. (2011).

**Epistemology and networking**

*What is the role of epistemology in networking?*

The claim that "the study of any didactic phenomena needs to question common epistemological models of mathematics" (Bosch et al., 2006, p. 1256) was present at CERME 4. Epistemological principles about how to interpret mathematical knowledge were discussed at CERME 6. At CERME 7, awareness of underlying assumptions of different theories also concerned the nature of mathematical objects (for example, Font et al. 2011). Epistemological aspects of theories were further developed in the WG. The construct 'reference epistemological model' was presented at CERME 8 to highlight this dimension (Ruiz-Munzón et al., 2013) and used at CERME 9 in Florensa et al.(2015). At CERME 9, we reflected on epistemology and networking of theories. Kidron (2015) analyzes how the epistemological analysis is tightly linked to the cultural dimension and

provides an example of networking that demonstrates how the social dimension might influence the epistemological analysis.

## 4. THEORY

The reader may be surprised to find a section on theory after the section on networking theories. The reason is that in all the work related to networking theoretical approaches, the notion of theory becomes crucial in many senses: what is a theory in mathematics education (from a local model to a well-established theory), how do people use theories, to what extent is 'theory' used as a metonymy for research activities including methodologies as intimately inseparable from theoretical perspectives, the use of 'metatheories' for networking, etc. The questions appeared as central ones in the first call for papers at CERME 4, where it was decided "to concentrate the discussion on research paradigms and/or theories within the context of their effect on empirical research" (Artigue et al, 2006, p. 1239). After the conference, a focus of discussion within the organizing team also centered on these issues, namely stressing two issues: "how to make explicit the level at which a theory operates" and "to have an awareness of the underlying assumptions of each theory" (*ibid.*, p. 1243). At all the conferences, some papers and many discussions dealt with these questions, making the reflection "reflective" on our own research work. This led us to re-examine some of the questions the diversity of theoretical approaches brings about.

*What is a theory in mathematics education? How do theoretical frameworks shape work in didactics?*

One of the first statements that came into play when comparing or just talking about theories is the heterogeneity of what is considered as a theoretical framework in MER and the consequent possible incommensurability of the investigations that are carried out in different theories. Diversity appeared as a main issue at CERME 5 (Arzarello et al, 2007) and at CERME 6, Prediger et al (2010) called for concrete discussions. Debates should avoid talking about the "essence" of theories but, instead, consider the role theories play in research methodologies, especially how they intervene in our delimitation of the "reality" we want to study, the kind of distinctions made and the way we question them. Differences soon appeared between approaches questioning the way students

learn/acquire knowledge and those starting from the kind of knowledge they are supposed to learn and the way this knowledge is made available through teaching, some of which were clearly stated during the work at CERME 7 (Kidron et al, 2011).

Behind the foci and implicit assumptions of theories are people – researchers – and collectives – research communities – that create, adapt, develop and share theories. This issue was raised in the plenary session on networking theories at CERME 6 (Dreyfus, 2010; Monaghan, 2010) and showed how the reflection on our own research work cannot be separated from the teaching and learning processes we look at. In consonance, some researchers made proposals for general models to describe the dynamics and the social dimension of MER, using for instance the notions of *semiosphere*, *shifts of attention*, *theoretical genesis* or *research praxeology*; this was discussed at CERME 7 (Kidron et al, 2011).

*How to extend the scope of a given theory to embrace a wider and more complex phenomenon such as the relationships between teaching/learning practices and research?*

This question was raised at CERME 5 and the papers by Goodchild (2007) and Jaworski (2007) provide interesting answers. Teaching and learning difficulties are the main motivation of MER and are thus usually located at the core of the construction of theories. Reciprocally, the influence of MER in teaching and learning practices is one of the main concerns of our community. "Exploring issues in networking theories is not a purely academic exercise; through this work we aspire to better understand the construction of mathematical knowledge" (Kidron et al., 2013, p. 2788). The interactions between MER and mathematic teachers' communities, in both empirical research and in teacher professional development, were analyzed (Goodchild, 2007; Jaworski, 2007). Some ambiguities in the use of theoretical guidelines for teaching appeared (Bingolbali & Bingolbali, 2015), as well as strong divergences (Roos & Palmér, 2015). Related to this, the issue of the use of tools or technologies used in teaching practices has also been a recurrent theme, not least because the implementation of new practices using digital tools raises new theoretical issues in our understanding of practice (Lagrange & Monaghan, 2010).

The role of different theoretical perspectives in approaching technology was also a topic of discussion (for example Cerrulli et al., 2006) and a plenary lecture on digital technologies as a window on theoretical issues in mathematics education was given at CERME 5 (Artigue, 2007).

*How do empirical studies contribute to the development and evolution of theories?*

Theories and empirical studies are intertwined and evolve together. Analyzing how a specific research paradigm influences empirical research and, vice-versa, how theoretical frameworks evolve through our empirical studies, appears to be an interesting way to deepen our research methodologies and our theoretical frameworks. The relationship can be shown in specific cases, for instance when two methodologies are applied to the same episode or set of data (Gellert, 2007). Workshop activities have also been proposed in some WGs to enable participants to interact with given empirical material or a spontaneous teaching problem (Prediger & Ruthven, 2007). However, it should be noted that empirical studies are designed with respect to the principles of a theoretical framework, so they may differ from one approach to another. Furthermore, the types of problems raised by an approach do not need to coincide with those of another. This can be seen in papers such as Bergsten (2007) on the limits of functions or Rodríguez, Bosch & Gascón (2007) on metacognition. The way to deal with a diversity of results, which might be contradictory in terms of practical implications, remains an open problem that should be addressed straightforwardly.

Even if it might appear as obvious, the work done at the WG constantly shows how what is called a 'theory' or a 'theoretical approach' in a given research vein is a dynamic entity that varies continuously. Different approaches present different degrees of development, from limited models to more extensive theoretical constructions. What makes them evolve and how this happens is also at the core of the methodological reflection within each framework and also in networking processes.

## 5. BEYOND ERME

**Ideas that originated in one geographical area, were brought into ERME and as a consequence were taken up more widely**

The important role of the theory is well demonstrated in the French school of *didactique des mathématiques*. The French school plays a particular role in European didactics. It offers two related well known theories: Theory of Didactical Situations (TDS) and the Anthropological Theory of the Didactic (ATD). Combining theoretical perspectives is not a new approach in the French school; since the eighties theoretical approaches have been connected within the French didactic community. Issues such as the important role of epistemology and the role of *a priori* analysis, which are strong in the French school, were brought into the discussions of the WG and were taken up more widely. We cite, for example, epistemological concerns which were highlighted as a consequence of the networking between TDS and the theoretical framework named Abstraction in Context (AiC). As a consequence of the networking experience between AiC and TDS researchers, AiC researchers decided to implement the idea of *a priori* analysis in an explicit way. Strong epistemological concerns in TDS were integrated in AiC in a way that reinforced the underlying assumptions of AiC.

**Ideas that arose within ERME and initiated or influenced development of mathematics education beyond ERME**

An initial idea that arose in the WG on theories is that *towards networking or dialogue between theories, researchers need to start working together on concrete cases*.

A call for collaboration was stated at the end of CERME 4:

> …as a research community, we need to be aware that discussion between researchers from different research communities is insufficient to achieve networking. Collaboration between teams using different theories with different underlying assumptions is called for ...    (Artigue et al., 2006, p. 1242).

As a consequence, a 'networking group' was created in which a group of European researchers collaborate since 2006. This Networking Theories Group aims to advance the networking idea as a research practice. Researchers looked at different aspects of a classroom lesson by means of different theoretical frameworks. Five theories were introduced in the networking process: APC, the theory of Action, Production and Communication; TDS, the Theory of Didactical Situations; ATD, the Anthropological Theory of the Didactic; AiC, the theory of Abstraction in Context and IDS, the theory of

Interest-Dense Situations. The results of eight years of intense collaboration between the members of the Networking Theories Group are described in the book *Networking of theories as a research practice in mathematics education* (Bikner-Ahsbahs and Prediger, 2014).

Another idea that arose in the WG is connected to the fact that theoretical frameworks are culturally situated. The different cultural background of each theory is a source of difficulty in the efforts of networking theories. Nevertheless, at ERME relationships between different theories were established as well as analyses of mutual insights offered by each theory to the other. In spite of the different educational context in which each theory was developed, networking was possible and rewarding. This is an important result that influenced, at least partially, the development of MER beyond ERME.

Since CERME 4 the idea of networking was further developed in different projects. Some of these projects might have been influenced from the work which was done at ERME. For example:

-The networking of theories was a topic of interest in two PME research forums in 2010.

-Methods and methodology in networking projects were further developed and some of them are described in the book *Approaches to qualitative research in mathematics education: Examples of methodology and methods* (Bikner-Ahsbahs, Knipping and Presmeg, 2015).

-In the ESM special issue Representing mathematics with digital media: Working across theoretical and contextual boundaries (Lagrange and Kynigos, 2014) different papers mention the work done in the CERME WG on theory .

- In some chapters in the book edited by Hodgson, Kuzniak and Lagrange (2016) *The didactics of mathematics: Approaches and issues (A homage to Michèle Artigue)* the CERME WG on theory was mentioned repeatedly.

**Ideas that arose within our WG and have become relevant for other WGs**

Relationships between different theories were discussed in other WGs. For example, in Dreyfus' (2010, p. 2) introduction to the plenary session at CERME 6 we read:

> The development and elaboration of theoretical constructs that allow research in
> mathematics education to progress has long been a focus of mathematics education

researchers in Europe. This focus has found its expression in many CERME working groups: some are focused around a specific theoretical approach and others allow researchers from different theoretical traditions and backgrounds to meet and discuss. For example, the working group on Argumentation and Proof at the present (CERME 6) conference has reported on passionate discussions about different theories and their relationships.

More recently, in the RME special issue *Institutional, sociocultural and discursive approaches to research in university mathematics education* (Nardi et al. , 2014) we read about research into the teaching and learning of mathematics at university level which deploys theoretical frameworks that are seen as increasingly essential in the field. The work is based on the work done at CERME within another WG, the WG on university mathematics education.  The selected frameworks are: ATD, TDS, Instrumental and Documentational Approaches, Communities of Practice and Inquiry and the Theory of Commognition.


## 6. REFLECTION ON WHAT HAS BEEN ACHIEVED AND A VISION FOR THE FUTURE

**What has been achieved?**

This question can be addressed in several ways. As academics our default approach is to look at the advancement of knowledge but we should also look towards the development of researchers, both novice and experienced researchers. Many people have participated in the six (at the time of writing) CERME theoretical perspectives WGs. Almost all of these participants have presented their work with follow up discussions. The 'social turn', mentioned in section 2 of this chapter, which sees thought as the product of social activity, applies to the development of 'theoretical thought' just as much as it applies to the development of mathematical thought. 'Theory' can be a frightening (austere) word to novice researchers and the act of joining a forum where theories are the object of discourse can help make theoretical considerations less frightening. This CERME WG has been a forum which has enabled this discourse. Some CERME WGs cease when they feel they have served their purpose but there is an argument that the theoretical perspectives WG should continue to allow this specialized discourse to continue. We now

turn to the advancement of knowledge and focus on two dominant foci in the preceding pages, epistemology and networking.

We expect that virtually everyone reading this could answer the question "What is epistemology?" with a response like "a/the theory of knowledge" but the question "What is your epistemology and how does it feature in your research?" is more tricky, even for seasoned theoreticians. The CERME theoretical perspectives WG has benefitted from the presence of representatives from the ATD and TDS at every meeting, to whom questions of epistemology are central:

> Pedagogy considers the knowledge to be taught as a given, and focuses on the best conditions or practices to teach and learn it: the knowledge is not problematic, the relationships of the students to it are (Chevallard, 2000). In contraposition, didactics locates the *epistemological problem* at the core of the analysis. (Florensa, Bosch & Gascón, 2015, p. 2635)

The impact of the focus on epistemology is reported in section 3 of this chapter in relation to the networking of theories, to which we now turn. Section 3 is the largest section in this chapter and this is fitting as the networking of theories has been the dominant theme of WGs to date. But we feel we are now at a more mature stage, recent work has focused less on strategies of networking and more on epistemological and methodological reflections:

In the last CERME we discussed cases in which the epistemological dimension permitted the networking. This was done, for example, by means of the idea of "reference epistemological model". In this paper, we notice how by means of networking, strong epistemological concerns in one theory might be integrated in another theory in a way that reinforces the underlying assumptions of this other theory. (Kidron, 2015, p. 2666)

### *A vision for the future*

We avoid predicting the future as this is a somewhat foolish activity as we are tied to our past experiences but, with this caveat, we can look forward to advances in unresolved areas of past work. The networking of theories deserves further attention in itself (the advancement of knowledge) and its potential in the field of didactics. We hope that the

meaning of networking will be extended, that members of future CERME theoretical perspectives WGs can use the experiences of the WG in networking theoretical approaches to facilitate other kinds of fruitful dialogues. For example, between mathematics education and disciplines close to mathematics education and between mathematics education researchers and mathematics educators who do not see themselves as 'theoreticians' but who nevertheless employ elements of theories in their instructional design. Such networking may enable a better understanding, from the perspectives of different theoretical frameworks, of the source of difficulties students and teachers encounter in their learning and teaching of mathematics. Such networking would, of course, involve 'the epistemological dimension' which we refer to above.

More attention could be given to networking and practice. We could also pay attention to the fact that we, as theoreticians, have a discourse/knowledge that purports to describe and explain practice but those whom we aspire to work with are likely to be outside of this domain of discourse. Mutual work to improve the teaching and learning of mathematics will likely result in new practices and new ways to describe and explain these new practices. New challenging tasks are offered as well by the dynamic character of theory especially when we face the changes of our cultural development as well as the changes of the context in which each theory emerged. Our final words express a hope for the future, that the CERME theoretical perspectives WG will continue to be a venue for open discussion with critical friends.

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