ERME book

Chapter 11: Technology and resources in mathematics education

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1. Technology group at CERME: history, background, aims and scope

The technology group at CERME was established at the very first CERME in 1999, as one of only seven themes, which highlights the importance that the mathematics education community places on the research on technology in mathematics teaching and learning. Over the subsequent conferences, the group continued to grow, from nine contributions at CERME 1 to around forty at CERME 6 and 7, which has led to the sub-division to two groups addressing the theme from the perspectives of students and teachers respectively.

From the outset, the group has considered mostly digital tools and technology encompassing mathematical software and applications, programming languages, communication platforms, and mobile devices. Recently, new conceptualisations of resources have led to tools and technology being considered more systematically as a component of the full range of resources available for students, teachers or teacher educators. Thus, since CERME 6, the group welcomes contributions not only on digital tools, but also on more traditional mathematical tools, textbooks and other resources.

Since the very first conference, the work of the technology group is framed by the "*three embedded levels*" considered "*when analyzing the use of tools in mathematics education*":

- "*the level of the interactions between tool and knowledge*
- *the level of interactions between knowledge, tool and the learner*
- *the level of integration of a tool in a mathematics curriculum and in the classroom*" (Laborde, Gutiérrez, Noss, & Rakov, 1999, pp. 183-184).

These three levels highlight the four components that can be distinguished in a didactic system involving a technology tool, namely a tool, some knowledge, students and a teacher, and possible relationships between these poles. Such a system can be represented by a "*didactic tetrahedron*" (Fig. 1) inspired from (Tall, 1986, p. 25). The didactic tetrahedron introduces a fourth component (vertex), a technology or a resource, into the traditional representation of a didactic system as a

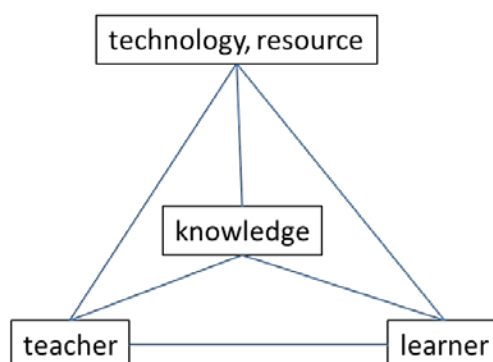triangle; teacher – learner – knowledge; and the impact of this introduction on the other three vertices.

Figure 1. Didactic tetrahedron.

The chapter aims to capture the work of the technology group community over time with respect to three sub-themes that have permeated its collaborative work: deep articulation of the nature of technological tools and resources and related interactions (Section 2); explanations of the principles and theories relating specifically to task design in technology-mediated environments (Section 3); and an expansion of our knowledge of theories and approaches that underpin and/or explain research (Section 4).

We conclude the chapter by summarising: the significant learning of the group in the past 20 years; our vision for the group's future organisation (and its influences on, and relationships with other working groups); and the possible trajectories of research that we anticipate. This is set within the context of a world where the rapid growth in both access to, and design of, new technologies within and beyond mathematics education is increasingly hard to understand.

2. Tools and resources

A *resource* or *medium* (lat.: medium = middle, midpoint) is something that is positioned between two domains. Language, gestures, paper, pencil, books, videos, ruler and compass, computer or interactive whiteboards are media and, in mathematics, they are positioned between mathematical objects (concepts, statements, algorithms) and human thinking, they mediate between mathematics and understanding. *Tools* are special media. Monaghan, Trouche and Borwein (2016) give a quite general and "*somewhat crude*" definition of a "*tool*" as "*something you use to do something*" (p. 5). This already shows the basic and global aspect of a tool in human activities, which makes it difficult to give a meaningful and satisfactory definition.

Mathematical tools allow us to create, to operate with and to change mathematical objects. Computers and especially programs like spreadsheets, dynamic geometry systems (DGS) and computer algebra systems (CAS) are *digital tools*. Mathematics education questions the meaning of media and tools for the teaching and learning of mathematics. The technology group of CERME is interested in the meaning of digital tools in the process of teaching and learning. In the following, we are not only going to reflect these discussions within the CERME technology group(s), but also connect the group's work with the global debate about digital technologies in mathematics and mathematics education.

Some common threads include:

- the recurring discussions on the relation between theoretical and practical aspects of tool use;
- the construction and development of theories adapted from existing theories and modified for (digital) tools and resources;
- the interrelationship between digital tools and other (non-digital) resources.

2.1 From suggestions of classroom use to more general reflections on technology-enhanced teaching and learning

Prior to CERME 1 in 1999, an intensive discussion about *new* – nowadays digital – technologies in mathematics education had already begun. At that time the main goal was to develop and evaluate strategies for the integration of digital tools in mathematics curricular and classrooms. For example, DGS had been in use for over 10 years; as one of the first DGS, Cabri-Géomètre had appeared in 1988. Consequently, the most important features of this class of tools: variation of objects by dragging; visualisations of loci; and macro-constructions; and the didactical implications of these; had been widely discussed. This included opportunities for new problem solving strategies; discovering geometrical theorems as invariants whilst varying objects (points, straight lines...); supporting conjecturing; and proving or discovering proofs.

CERME 1 built on this earlier work by adopting a research-oriented view with questions such as: What are students' views and interpretations of dynamic tools? What is the relationship between drawings and symbols? How can technologies support the learning of the concepts of variables and functions? The contributions to this conference concentrated mainly on particular uses of tools in the classroom with many examples of the dynamic affordances of these tools.

The next two conferences, CERME 2 and 3, brought a shift from the fascination of the technological possibilities, and from suggesting examples of tool use in special content-oriented environments, to theoretical reflections concerning comparisons of tools and the relations of, and connections between tools. Moreover, other aspects of technological tools appeared, such as technology use in distance education and the impact of technology on the nature of examinations. In this period increasing attention was being paid to teachers' roles in technology-mediated mathematical activities and their associated knowledge base.

In this period of time the focus of the scientific discussion concerning digital technologies *outside CERME* was CAS. In 1995 the first calculator with CAS was available (the *Texas Instruments' TI-92*), followed in 1999 by the *Casio FX 2.0*. These calculators generated big expectations in the community. The possibility for students to have a ready and available tool, which would do (nearly) all symbolic transformations of high school mathematics by only pressing a button, was predicted to lead to deep, far-reaching changes for the content of the curriculum and its examination. These particular tools promoted the research community to rethink existing theories and develop new interpretations (See Section 4).

By the time of CERME 4, the questions concerning tool classification and design were more specified. It was evident that the word *tool* meant a variety of objects with different characteristics. There were special *pedagogy-free environments* like CAS, DGS, graphing and programming tools. Other tools, often called applets, microworlds or special learning programs could be considered as local *dedicated* environments. This raised the question concerning the characteristics of each of these tools: What kind of technological tools do we need in our teaching? Moreover, it was also apparent that the reflective use of tools in the learning process needed theoretical frameworks specific to the tool and mathematical content, for which the *instrumental approach* emerged as a central theoretical framework. CERME 5, drew on the concepts introduced by the instrumental approach and opened questions about the design and appearance of tools to support successful appropriation, integration and institutionalization for both students and teachers. These ideas are described further in Section 4.

2.2 The move to technologies as tools within a resource system

In the beginning of the new century, there were two requests or claims concerning the use of technology in the teaching and learning process. On the one hand there was a request for more

sustained and longitudinal projects to obtain significant and convincing research findings in real classroom situations. A number of empirical studies had taken place over longer period that had revealed results that were common to many studies. e.g. e-CoLab[1] project in France, RITEMATHS[2] project in Australia, or the 'M[3]—**M**odel Project New **M**edia in **M**athematics Education' (Weigand, 2008).

These increasingly robust findings established that using technology provides opportunities to:

- work within dynamically-linked multiple mathematical representations;
- construct new problem solving environments;
- design more personalised learning;
- integrate more realistic modelling problems into the mathematics classes;

whilst highlighting that technology use demanded:

- new types of learners' knowledge, e.g. to move between representational forms with understanding;
- new types of teachers' knowledge in relation to design, implementation and assessment;
- some rethinking of the content and hierarchies of the mathematics curriculum and its assessment.

On the other hand, questions around connectivity were emerging, e.g. how to connect students and mathematics through technology, students and teachers, and technology to other resources for teaching and learning (Monaghan et al., 2016, p. 433). On the tool level, CERME 6 reacted to this aspect by adding the word "*resources*" in the name of the TWG, which was previously called "*Tools and technologies in mathematical didactics*". This expressed the need for considering technologies within a range of resources available for students, teachers and teacher educators. Resources might be software, computers, interactive whiteboards, online resources, but also traditional geometry tools and textbooks. This demanded a deeper understanding of the relationship between technologies and the traditional tools and resources. How can these *old* and *new* resources interact with each other? For example, how can digital features be incorporated in new forms of textbooks? However, in the subsequent period there have been only a few

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[1] e-CoLab = Expérimentation Collaborative de Laboratoires mathématiques, see http://educmath.ens-lyon.fr/Educmath/ressources/lecture/dossier_mutualisation/ecolab.pdf

[2] RITEMATHS = The project is about the use of real problems (R) and information technology (IT) to enhance (E) students' commitment to, and achievement in, mathematics (MATHS). http://extranet.edfac.unimelb.edu.au/DSME/RITEMATHS.

contributions to the CERME technology group that have addressed this demand that mainly referred to textbooks, 'e-textbooks' and some online courses.

2.3 From the students' uses of tools to that of teachers

The introduction of the term *resources* also was in line with the development of a *documentational approach* of didactics (elaborated in Section 4). This places a greater emphasis on the roles and actions of teachers, a theme that was first introduced at CERME 3 when it posed the following questions:

- How can we understand how mathematics teachers integrate technology in their teaching?
- How might we encourage more mathematics teachers to use technology?
- How does using technology change the ways mathematics teachers think about teaching and learning? (Jones & Lagrange, 2003).

These questions, supplemented by others, have remained an important focus for the group, and sowed the seeds for the more recent sub-division of the group (see Section 1). However, with respect to the tools, this has meant that, in many cases the user is no longer the student alone - the teacher perspective is now not only considered alongside, but specific functionalities developed solely to support the teacher.

If we take more holistic perspective of tools that includes teachers, we should consider all of the processes for designing teaching: looking for resources, integrating these in a personal resource system, implementing in practice, sharing with colleagues, revising to take account of feedback, etc. This wider discussion concerning teachers' integration of technology continued at CERME 8 through particular exemplifications: for example, using interactive whiteboards in geometry; creating tests and examinations in a CAS-environment and exploring the potential of technology for the teaching and learning of functions.

In addition, the group again emphasized the need to focus research more intensively towards longer-term studies involving *ordinary* teachers and *ordinary* classrooms settings. Another request was to concentrate more on emerging research themes present within the general technology literature, which at that time had been underrepresented at ERME conferences. Examples included the design and use of innovative technologies such as Web 2.0, mobile

technologies, the development of e-textbooks or the design and use of technologies and resources for learners with special educational needs.

2.4 From local empirical studies to scaling up good practices with digital resources

In 2010 the 17th ICMI study "*Mathematics Education and Technology – Rethinking the Terrain*" (Hoyles & Lagrange) was published, revisiting the theme of the very first ICMI study "*The Influence of Computers and Informatics on Mathematics and its Teaching*" (Churchhouse 1986). Given the great enthusiasm for the new possibilities that computers and technology might open to mathematics and mathematics education 20 years ago, the 2010 study gave a disappointing account of the current situation concerning the dissemination of technology. Despite a high number of research studies and accounts of classroom practices, the use of technologies in mathematics education and the impact on curriculum and assessment change was still limited.

The group at CERME 5 had also first highlighted the predominance of small empirical studies, calling for longer-term larger-scale research. More recent ERME conferences have begun to include contributions on this theme (Clark-Wilson, Hoyles and Noss, 2015, Lavicza et al., 2015), which explore ways to implement or transfer ideas or consequences of empirical investigations concentrated in the word *scaling* or *scaling-up*, i. e., researching how to realise the results of research into the reality of daily teaching.

3. The design and implementation of digital mathematical tasks

Mathematical tasks are an integral element of mathematics education and its associated research agenda. The design of *digital* mathematical tasks has recently received particular attention, as a sub-theme of the 22nd ICMI Study "*Task Design in Mathematics Education*" (Watson & Ohtani 2015) and within a dedicated volume of the "*Mathematics in the Digital Era*" book series (Leung & Baccaglini-Frank 2016).

This section focuses on the design of technology-mediated tasks with an emphasis on the explicit design decisions that influence how tasks are subsequently used in and for mathematical learning. It considers how such tasks are combined or developed to produce learning sequences or *courses* and finishes by addressing an important emerging theme - the design of tasks for prospective and practising teachers/lecturers for *their professional development to introduce and use technology in mathematics classrooms*.

3.1 Tasks and task designers

At the first meeting of the TWG, it was highlighted that:

> One of the key issues for teachers is how to design tasks based on tools or technologies in which real questions for the learner emerge from the use of the tool, in which the tool is relevant and gives a new dimension to the task. (Laborde et al., 1999, p. 187)

Indeed, elements of task design have featured within many contributions to the early conferences, varying from the individual design decisions for tasks within classroom or research laboratory settings to those concerning whole courses within large-scale university courses. However, more usually the tasks were offered as a *given*, often subsumed within the notion of the tool or activity. Consequently, given the length and scope of a CERME research paper/presentation, unless the research was specifically reporting aspects of task design, there was little opportunity for the task design to be explicitly described or theorised about. Early collaborations within the technology group at CERME highlighted the possible gap between a task designer's intended learning goals for a digital task and the mathematical meaning that learners ultimately construct. Whilst it has always proved challenging to separate aspects of the design of the tool from that of the task itself, in this section we try to distil the contribution of ERME research to the community knowledge concerning task design.

The term task designer has always held a broad definition to include: teachers; researchers; teacher educators; and technology developers – with many of the CERME participants representing one or more of these roles – and, as a result, offering enriched perspectives. Task designers appear to have been motivated by two broad approaches:

1. Development of innovative technological tasks that provide access to traditional mathematical knowledge and activity – often attempting to create a technology-mediated version of the equivalent paper and pencil task. For example, an early paper at CERME 2 by Gélis and Lenne (2001) described the design of tasks using CAS-based technology to support upper secondary French students to learn about arithmetic sequences.

2. Development of innovative technological tasks that may lead to new forms of mathematical knowledge and activity. For example, the advent of dynamic geometry software and the dragging affordance led to a re-examination of the role of empirical measurement and 'checking by dragging' within the processes of justification and proof. (e.g., Olivero & Robutti, 2001). By

contrast, researchers who have been involved in the design of digital environment that have aimed to disrupt mathematics education norms by offering new ways of mediating existing mathematical knowledge or suggesting new epistemologies have concluded that resulting tasks lack educational legitimacy at a system level. From the perspective of teachers and schools, tasks may not align with institutional constraints such as the prevailing classroom norms or assessment regimes, which may not have kept pace.

However, it has been common for research that began with more pragmatic motivations to integrate technology to improve the learning experiences in relation to the traditional curriculum to report findings in relation to new epistemologies and learning hierarchies.

3.2 Contexts and theories for task design

Reviewing the earlier contributions to the technology group, the vast majority of papers included examples of tasks that were used in the context of the research or study. However, it was rare for authors to describe explicitly: their motivations for the design of the task; choice of representational forms; and their intended mathematical progression of potential pathways through the task. For example, Jones (1999) described an empirical study in which pairs of students worked through a sequence of "*specially designed tasks*" involving the construction of quadrilaterals using Cabri-géomètre over a nine-month period (See Figure 2).

The Square
Construct these figures so that they cannot be "messed up".

What do you know about this shape from the way in which you constructed it?

Think about ..... sides
diagonals
Explain why the shape is a square.

Figure 2 A task asking pupils to construct a square that is invariant under drag (Jones 1999. p. 258)

Interestingly, the CERME technology group seems to have arrived at some common understandings of what is inherent in such *specially designed tasks* in that they are often constructivist in nature - allowing students to explore and create mathematical knowledge, often working in pairs or small groups. It is only since CERME 8 that a number of researchers have begun to share their analyses of aspects of task design using theories such as Brouseeau's theory of didactic situations (Lagrange & Psycharis, 2013; Mackrell, Maschietto & Soury-Lavergne, 2013) and Variation theory (Attorps, Björk, Radic & Viirman, 2013).

3.3 From individual task design to the design of courses

The complex design decisions related to the development of a university-based numerical methods course were described by Belousova and Byelyavtseva (1999) at CERME 1, who raised important questions about the balance between: students' empirical and theoretical work; individual work and group work; and a range of mediational roles for the technology. These themes have recurred in many subsequent contributions to the technology group.

Task design within digital environments incorporates an articulation of the way in which the task is initiated/mediated. Given that a potential role of technology is to support the communication of both mathematical and meta-cognitive knowledge between the student and 'teacher', there have been many CERME contributions that describe research in which aspects of the teacher's role is outsourced to the technology in the form of scaffolding and feedback to the learner. The notion of *e-learning* can often imply a learning pathway mediated in the absence of an obvious *teacher*. For example, at CERME 8, Fredriksen (2013) reported on design decisions concerning prospective teachers' uses of video lectures within an online course in Norway, whilst at CERME 9, Jančařík and Novotná (2015) researched how teachers scaffolded student learning in an online course for talented children in the Czech Republic, concluding the importance of *offline discussions*.

3.4. Implementing digital mathematical tasks in research and classroom settings

Whilst it is easy to conceive that the task is an artefact that is offered to learners, a crucial component of task design relates to the many decisions about when, how and with whom a task is implemented in a research or classroom setting. Whilst this detail is a fundamental component of any research methodology, it is often an under reported aspect of task design. For example, the research reported by Gallopin and Zuccheri (2001) included detailed description of the phases of the "*didactical path*" adopted for their study that used two contrasting dynamic geometry softwares with Italian secondary school students.

3.5. Designing digital tasks for prospective and practicing teachers

More recently, attention in the technology group is shifting to research that concerns the design, implementation and impact of tasks that are intended for prospective and practicing teachers for their professional learning concerning technology use in classrooms. The group has always acknowledged that teachers needed to undertake specific professional learning to achieve this aim, however the nature and complexities of this learning have often been under-defined. Since CERME 7, a number of theories have been developed that articulate aspects of teachers' technological knowledge and practices, see for example Ruthven (2009), Drijvers (2011), Haspekian (2011) and Rocha (2015). However, research on the application of these theories within the design of tasks intended for teachers' professional learning initiatives is still in its

infancy. A number of European studies are beginning to address these gaps, within a wider focus on scaling technology use and will report at CERME 10 in Dublin.

4. Theories and approaches concerning technology and resources

Theoretical frameworks became an explicit theme discussed within the technology group since CERME 3. This was a clear milestone in the progress of the group. In what follows, we outline the variety of theories and approaches that have permeated the group and trace the evolution of theories used in research on technologies in mathematics education over the past 20 years.

In charting the progress of the group since this time, the considerations of theories presented below are organized with reference to the didactic tetrahedron, attempting to isolate faces or edges according to the research focus to gain a deeper insight into the strengths and limitations of these frames, although we are aware that in technology-supported teaching/learning situations, all four vertices are intertwined and interact with each other.

4.1 Technology and knowledge

Prior to CERME 3 most of the contributions investigate "*a new epistemology of mathematics created by the use of the technology*" (Laborde et al., 1999, pp. 185-186). Referring to the didactic tetrahedron (Fig. 1), these issues are related to the *technology – knowledge* edge and address mostly the epistemological dimension of the use of technology in mathematics education. They can be classified in two categories:

(1) *Exploring how technology mediates knowledge and the consequences of this mediation on the knowledge itself*. A paradigmatic example is provided by the new behavior of objects in a dynamic geometry environment, which actually gives birth to a new kind of objects (Laborde et al., 1999, p. 185). A consequence of interacting with such new objects on the students' conceptualization of linear algebra notions is explored by Dreyfus, Hillel and Sierpinska (1999).

Knowledge mediation by technology can be addressed in terms of the *computerized transposition* (Balacheff, 1993) bringing to the fore tool-designed constraints (internal, command, interface) introduced by the use of a computer likely to impact upon the mathematical knowledge at stake. Alternatively, knowledge mediation can be interpreted through the notion of the *epistemological triangle* (Fig. 2), which represents "*the connection between the mathematical signs, the reference contexts and the mediation between signs and reference contexts which is*

influenced by the epistemological conditions of mathematical knowledge" (Steinbring, 2006, p. 135). Both theoretical constructs are usually combined with an *epistemological content analysis*, which defines the essence of the mathematical knowledge at stake.
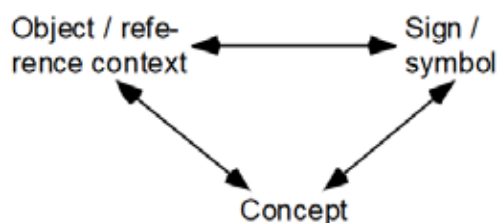

Figure 2. Epistemological triangle.

(2) *Investigating better ways to learn mathematical concepts with technology*. This category of research focuses on software or task design and on *a priori* analysis of the potential of the task as instrumented by a chosen technology in order to achieve a known learning goal. Among the frameworks mobilized by the researchers are:

- *semiotic mediation* (Bartolini-Bussi & Mariotti, 2008) assuming that, in social contexts, mathematical meaning can be created from specific uses of a tool (e.g., dragging in a DGS can be associated with understanding a DG construction as a figure rather than just a drawing);

- *situated abstraction* (Noss & Hoyles, 1996) used to "*describe how learners construct mathematical ideas by drawing on the webbing of a particular setting which, in turn, shapes the way the ideas are expressed*", the situatedness emphasises the "*specificities of the situation, and in particular [...] the linguistic and conceptual resources available for expressing mathematically within them*" (p. 122).

4.2 Technology, knowledge and student(s)

From the first ERME conferences, researchers investigated "*the complex interplay between the work in a technological environment and the development of mathematical understanding and skills*" (Barzel et al., 2005, p. 928), addressing the cognitive and instrumental dimensions of the use of technology.

Several theoretical constructs are used to explore such interplay:

- The concepts of *embodied cognition* and *metaphors* (Lakoff & Nuñes, 2000) viewing mathematical meaning as rooted in our experience of common phenomena such as

movement and working as a metaphor. This framework is used for instance to analyze students' cognitive processes while doing activities involving artefacts such as movement sensor and the corresponding conceptualization of function.

- The *theory of didactic situations* (Brousseau, 1997) considering digital technology as a component of the 'milieu' with which a learner interacts and allowing analysing possible learner – milieu interactions and the related learning outcomes.

- The notion of *instrumentation* appears at CERME 3 as the approach that "*distinguishes the instrument (a psychological construction) from the artefact (the material object involved in an instrumented action)*" (Jones & Lagrange, 2003). The *instrumental approach* (Rabardel, 2002), which emerged as the most central theoretical framework at CERME 4 conference and subsequently, pinpoints that "*given a tool, the genesis of a fruitful instrument is far from self-evident, but is the result of a social process, guided by a set of tasks in a given institution*" (Barzel et al., 2005, p. 929). This construct has proved particularly helpful when studying long-term uses of technology.

4.3 Technology, knowledge and teacher

As previously alluded to, the awareness of the importance of a teacher dimension in research on technology in mathematics education (the *technology – teacher – knowledge* face of the didactic tetrahedron) emerged slowly. It was first considered explicitly during CERME 3 and also featured in a small number of contributions at CERME 4, as pointed out by Barzel et al. (2005) who commented, "*On the issue of the second theme, the role of the teacher in technology-rich mathematics education, we observe that in spite of the relevance that is attributed to this theme, little research was reported in this working group*" (p. 937). Subsequently, four aspects of the teacher dimension have been addressed by the technology group community:

- Investigating the role of the teacher in a technology-based setting.
- Analyzing teachers' practices in using technology.
- Characterizing the new knowledge and skills required for an efficient use of technology.
- Designing and assessing teacher education/teacher training programmes.

The interest of researchers in the *instrumental approach*, used so far for studying issues related to the *knowledge – technology – learner* face of the didactic tetrahedron, raised the question of

its adaptation for teachers. Soon, new concepts have been developed and shared within mathematics education community through, in particular, the CERME technology group:

- *instrumental orchestration* (Trouche, 2004) defined as "*the intentional and systematic organisation and use of the various artefacts available in* [...] *computerised – learning environment by the teacher in a given mathematical task situation, in order to guide students' instrumental genesis*" (Drijvers et al. 2009, p. 1350);

- *double instrumental genesis* (Haspekian 2011) highlighting that, in order to efficiently use a digital tool in her teaching, a teacher has to develop not one, but two instruments: a *personal instrument* for mathematical work and a *professional instrument* for teaching mathematics.

- The *documentational approach to didactics* (Gueudet & Trouche, 2009), drawing on the instrumental approach, substitutes artefact by resource and instrument by document and shows a great variety of resources intervening in teachers' 'work: textbooks, students' worksheets, Internet resources, discussions with colleagues, etc. The development of this framework has led to the re-conceptualization of tools and technology within the CERME group by considering them within a wider range of resources (see Section 2.2).

Several studies aim at analysing teachers' practices with the use of technology in order to get a deeper insight into the complexity of technology integration. Assude (2007) elaborates a theoretical tool allowing characterizing the degree of teachers' integration of technology taking into account both the instrumental and the praxeological dimensions. Abboud-Blanchard and Vandebrouck (2013) have developed a model for studying the evolutions of teachers' practices in terms of technology uses, combining the *activity theory* (Engeström, 1999), the instrumental approach and the *double approach to teachers' practices* (Robert & Rogalski, 2005). The latter considers a teacher's activity through five components (*personal*; *mediative*; *cognitive institutional* and *social), a* frame also used by Emprin (2007) to analyze training courses aiming at the use of technology.

The *Technology, Pedagogy and Content Knowledge* (TPACK) framework (Mishra & Koehler, 2006) is a dominant frame used to address teachers' professional knowledge and skills, suggesting seven categories of this knowledge: mathematical content knowledge (CK), pedagogical knowledge (PK), technological knowledge (TK) and all possible intersections of

these (PCK, TCK, TPK and TPCK). However, to date, only a few studies within this frame have been reported to the group.

The notion of *community* is another concept widely used, mainly in relation with teacher professional development. *Communities of practice* (Wenger, 1998), or *communities of inquiry* (Jaworski, 2005) are either established purposefully by the researchers or teacher trainers to accompany teachers' efforts with integrating technology in their everyday practice (e.g., Fuglestad, 2007), or they develop spontaneously around Web2.0 tools enabling sharing resources and practices (Trgalová, Jahn & Soury-Lavergne, 2009).

This overview of theories used in CERME contributions reveals a wide variety of frames, which can be seen as a wealth of the research field, but there is a risk of "*the framework compartmentalization that could hinder the capitalization of knowledge and its practical exploitation*" (Artigue, Bosch, & Gascon, 2011, p. 2381). The awareness of this risk appears quite early within the CERME technology group, at CERME 4 that concluded, "*a more ecological and systematic approach is needed rather than a unifying theory, which takes into account the existing subsystems, and which combines various theories focusing on each of these subsystems (didactics, instrumental approach, situated and distributed cognition, community of practice)*" (Barzel et al., 2005, p. 929). The subsequent conferences call for further development of theories toward a comprehensive and more coherent landscape of articulated frames.

5. Conclusion

In concluding this chapter, a key question to ask is, what do we know now about technology integration in mathematics teaching and learning that we did not know before the technology group was established? The collective knowledge and experience of the community would suggest the following conclusion. It takes a significant amount of time for learners and teachers to become fully instrumentalised, that is to learn to use and apply the technology for their relevant mathematical purpose, which for teachers includes important didactic considerations and the development of their resource systems. The ongoing innovation of technological tools, within and outside of mathematics education, requires us to continue to address the *learner-knowledge-technology* face of the didactic tetrahedron. This might mean that we appear to reinvent the wheel again and again, as new researchers, teachers and technology designers encounter for the first-time *known* issues and challenges concerning technology integration for

mathematics education. However, as goals in education have to be continually rethought and evaluated due to new developments in society, science and education, this also suggests it may not be possible for knowledge of theory to short-cut this process and it is a key role for the community of the CERME group to support important connections to be made. In the beginning, the focus of the research was on the effects of using technology on students' learning and teachers' practices. Now, as we know more about these effects, our attention has shifted to be concerned with researching how we can scale 'successful' innovations in mainstream education systems.

It is notable that technology is now a visible element across all CERME groups. This is an indication of how it now permeates and has wide legitimacy across the mathematics education research landscape. This leads us to question our special role and to justify the important questions or topics that we can make a distinct contribution to. In some ways, this justification comes from the continued disappointment in the lack of widespread uptake of technology, first highlighted at CERME 5 thus "*The use of technologies has simply not scaled up and the changes promised by the case study experiences have not really been noticed beyond the empirical evidence given by the studies themselves*" (Kynigos et al., 2007, p. 1541). It is the technology group that perseveres to confront this issue and, although new, possibly more exciting technologies arrive on the scene with great promise to revolutionise classrooms, we find ourselves expanding our experiential and theoretical knowledge in order to be able to better inform future attempts to scale.

One challenge faced by the group is that it has grown from only nine accepted paper submissions at CERME 1 to thirty-nine accepted paper submissions (and eight posters) at CERME 9. The TWG leaders responded by dividing the group to emphasize research that foregrounded students from that which foregrounded teachers. However, given the frame of the didactic tetrahedron this is somewhat dissatisfying as most studies feature both perspectives. An alternative might be to divide the group by educational phase, as in the ICME conference series. However, we question whether it is possible to partition in a wholly satisfying way and this will be an ongoing topic for discussion for the group.

There is still a need to develop more comprehensive theoretical frameworks to address *old* but still topical themes, such as task design and methods for large-scale dissemination of *research-*

informed practices with digital technologies. The role of technologies within processes of: formative assessment; of networked classroom technologies; and e-learning (particularly *Massive Open Online Courses*) appear among the emergent issues that require further theoretical and methodological development. It is worth noticing that this issue becomes shared with the CERME group on theoretical perspectives and approaches as its call for contributions at CERME 10 includes "*Theories for research in technology use in mathematics education*"[3].

Finally, with a focus on emergent technologies, we anticipate that future CERME conferences will feature research on *touch screens and human-computer interaction,* which is already a frequently discussed topic in present classrooms questioning how gestures can help visualising and, hopefully, understanding mathematical concepts; *3D technology*, including the use of 3D printers within mathematics education; *virtual and augmented reality* in mathematics education; *artificial intelligence* features to include intelligent tutoring and support systems that take account of large data sets; *ICT-support and special needs students,* particularly students with physical disabilities; and *digital technologies that support individuality*, for example, the creation of portfolios and personalised e-textbooks.

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[3] See http://cerme10.org/scientific-activities/twg-teams/

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