Research on University Mathematics Education

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1. Introduction

Mathematics is an ancient scholarly discipline, with distinct and related practices of *research* and *education*. CERME is concerned with one such practice: *research on mathematics education* (RME), which is the study of mathematics teaching and learning at all levels, including university mathematics education (UME). And that, *research on university mathematics education* (RUME), is the topic of this chapter. [Note: the abbreviation RUME is also used by the American association SIGMAA-RUME, but with a slightly different meaning: *undergraduate* mathematics education.]

Internationally, RUME has emerged along with RME in the course of the 20th century, beginning with the efforts of Klein (1908) to develop stronger ties between UME and mathematics education in schools. In terms of systematic research, early milestones for RUME appear in the volume on *Advanced Mathematical Thinking* (Tall, 1991) and the 11th ICMI study (Holton, 2001). The first six CERME each had 10-20 papers on RUME in different working groups; at CERME 4, 5 and 6, most RUME papers appeared in the working group "Advanced Mathematical Thinking". A group explicitly focusing on RUME has been present since CERME7 in 2011, and the number of papers has increased from 24 in 2011 to 49 in 2017.

In this chapter, we present selected main problems and results from RUME which have appeared at CERME, with a deliberate focus on current and emerging trends. The chapter has two main parts: research into current or "normal" practices of UME (with no direct intervention), and developmental or experimental research, where an intervention design is part of the research project. We have thus chosen to concentrate on *problems* and *results* and leave *methods* and *theoretical paradigms* more in the background (for the latter, we refer to the special issue edited by Nardi et al. (2014), largely based on CERME work on RUME).

In the last, concluding section, we draw up some possible directions for the future of RUME and in particular, its potential contributions to UME. As the preceding selection and outline of work from CERME, these visions are naturally shaped and limited by the perspectives of the authors.

2. Mathematics education at University: What is it?

This first section focuses on *descriptive* research, i.e. studies of UME "as it is" (no intervention). It is structured according to *foreground research objects*: contents; methods and resources; transition phenomena; student experiences; and teaching non-mathematics specialists.

2.1 Mathematical content in university programs

As in all areas of RME, it is important within RUME to be attentive to the variations of cultural and institutional contexts of the phenomena we study. We notice, however, that the *mathematical contents* taught and learned is a remarkably stable and recognizable factor in many studies. A large portion involve Calculus and Linear Algebra, which are indeed commonly taught during the first year of university programs, both in the mathematical sciences, natural sciences, engineering and business (cf. Adams, 2002). Many papers study specific challenges students face with notions such as limits and derivatives (e.g. Hähkiöniemi, 2005) or infinite series (González-Martín, 2013); in these and other studies, it is found that students may succeed quite well with institutional (computational) requirements related to such notions, and still have little or no idea of their theoretical and practical significance.

Studies involving more advanced domains from pure mathematics, such as abstract Algebra or Topology, slowly but surely begin to appear in CERME papers (e.g. Hausberger, 2015). We also increasingly find studies carried out in settings where the content is not classical mathematics, such as an analysis of modelling within an introductory course on electrical engineering (Biehler, Kortemeyer and Shaper, 2015), and the ways in which knowledge from high school mathematics acts as a resource or obstacle for students in such a course. An important insight from such research is that the uses and meanings of mathematics at university are not simply confined to courses with a mathematics label.

2.2. Methods and resources in UME teaching

The investigation of teaching methods, both new and 'traditional', is a constant theme in RUME. For instance, lectures are a common format in UME, unlike other levels, and Bergsten (2011) studied Swedish students' views of this format. He observes that the students value the lectures, for instance because they consider them as helpful to have an idea of what is expected at the exam. They also like the explanations given by the lecturer, both intuitive and more formal ones, and consider them useful to learn proofs. They appreciate their lecturer's pedagogical awareness.

The effects of new technology in UME have been studied rather extensively, just as at other levels of teaching. In some cases the technology considered is specific to the university level, in particular concerning distance learning (Misfeldt and Sanne 2007). In more recent works, technology is considered as belonging to sets of resources of different natures. Gueudet (2013) analyses how a teacher in a technological institute designs his own resources, using in particular a computer algebra system (Scilab) and develops a structured resource system, according to his professional knowledge and beliefs.

Recent works have also taken a closer look at assessment practices in UME. Thoma and Iannone (2015) analysed three examination tasks by applying the 'Mathematical Assessment Task Hierarchy' by Smith et al. (1996), that focuses on knowledge and skills, and a discourse orientated framework by Morgan and Tang (2012) that is based on systemic functional linguistics (Halliday, 1978) and Sfard's (2008) theory of commognition. Both approaches allow for identifying various complementary and implicit characteristics of tasks and, potentially, the "learning approaches" they promote.

A special characteristic of UME is that many of the teachers are research mathematicians, and research presented at CERME conferences has drawn on different forms of interviews with mathematicians. Burton (1999) interviewed seventy mathematicians about their views on mathematics and on their teaching; this study is an example of a CERME paper which was later developed considerably, to result in an influential book (Burton, 2004). Burton's results drew a worrying picture. For example, mathematicians acknowledged the existence of different *thinking styles* in mathematics, and that they themselves had a dominant one. However, they did not seem to consider that their own thinking style influenced their teaching, and did not try to cater for students' different thinking styles in their teaching. Subsequent works lead to more positive outlooks. For example, Nardi and Iannone (2005) demonstrate that mathematicians can have a rich reflection on their students' mathematical practices, and be interested in engaging in teaching innovation. Similarly, Mesa and Cawley (2015) analyze the learning processes of university teachers who engaged in a project on Inquiry Based Learning. Such projects seem to share the aspiration of Burton (1999) for mathematics taught at university with "approaches not too dissimilar from those of research mathematicians" (p. 98). The tendency that UME teachers are increasingly engaging in efforts to this end is confirmed by several other papers presented at CERME, and we return to this in Section 4.

2.3 The transition from school to university

The transition from school to university has been an important topic in all CERME, starting from CERME 1, then developing in the Advanced Mathematical Thinking (AMT) group and finally in the UME group. We notice a central evolution in the theoretical frames: cognitive approaches, which were very present in the AMT group, seem in recent years less frequent, while socio-cultural, discursive and semiotic approaches developed. We claim that this evolution has two main consequences.

The first one concerns the focus of the studies. Many changes happen during the secondary-tertiary transition; the choice of a specific change as focus is strongly linked with the theoretical perspective. In the AMT group, many papers focused on *conceptual change*. Biza, Souyoul and Zacharides (2006) showed that novice students create synthetic models, mixing their personal beliefs and the scientific theory. Studying these models in the case of the concept of tangent to a curve, they observed that the students developed a synthetic model incorporating the properties of circles' tangents, hindering the development of an adequate concept image of the tangent. Using a socio-cultural approach, namely the Anthropological Theory of the Didactic (ATD), Winsløw (2006) chose a different focus: the changes in the *praxeologies*. He identified two kinds of such changes: (1) from praxeologies at secondary school centered on tasks and techniques, to "full praxeologies" including technology and theory at university; (2) from familiar tasks and techniques to tasks and techniques of a more theoretical nature. This second kind of transition can happen later, for example from the first to the second university year. De Vleeschouwer and Gueudet (2011) studied changes in the *didactic contract*, between secondary school and university. They noticed different kinds of changes: very general changes, concerning for example personal work; changes related to mathematics as a discipline, like the expectations in terms of rigor; and changes related to a specific mathematical content, in their case duality in Linear Algebra. More recently, authors study changes in the mathematical discourse. Petterson, Stadler and Tambour (2013) analyse students' understanding of function as a 'threshold concept' (an initially troublesome concept which leads to a new understanding), using the commognitive approach; they identify evolutions and stabilities in the discourses of the students. In these and other cases, we can observe how theoretical framings of transition phenomena, initiated at one CERME, are subsequently taken up and developed by other authors in later conferences and beyond.

The second, strongly related, kind of evolutions concerns a broader view on secondary-tertiary transition conveyed by the research works. The view on transition has evolved from a local cognitive change, taking place at the beginning of university, to a long-term social and cultural process developing over one or several years. Most of the early studies concentrate on the difficulties with particular mathematical contents met by students entering university. For Linear Algebra, Dorier, Robert, Robinet and Rogalski (1999) argued that the novice students fear formalism. Nevertheless, according to their historical-epistemological analysis, formalism should not be avoided: on the contrary it must be put forward, but introduced as the answer to a problem. Studies of local difficulties are still present: Vandebrouck (2011) showed that three perspectives can be adopted on functions: a point-wise perspective, a global perspective and a local perspective. At university, it is necessary to master these three perspectives and to be able to combine them; but freshmen have difficulties with the point-wise and the global perspectives. The socio-cultural approaches mentioned above invite to study transition as more long-term processes. For example Stadler, Bengmark, Thunberg and

Winberg (2013) followed the approaches of students to learning mathematics along the first university year. They showed that students progressively rely less on the teacher, and more on peers or Internet resources.

2.4 Students' experience of UME

Over the years, research into students' experience of university mathematics presented at CERME has gained scope, substantively (in terms of the areas this research examines) and theoretically (in terms of the theoretical underpinnings of this research). In what follows we trace this growth from studies of student learning of particular mathematical topics and aspects of mathematical thinking such as proof and proving, to include also studies of student affect – including attitudes, motivations and emotions – as well as studies of broader institutional and social issues, such as recruitment and retention. Much along the lines we describe in 2.3 in relation to studies of transition, the broadening of substantive scope has occurred in tandem with a broadening of theoretical scope. We dare say that *it is in the study of student experience of university mathematics that the shift from the AMT working group* (dominated by what we labelled in 2.3 as cognitive studies of mathematical learning) *to the UME working group* (currently populated with a very diverse set of studies that endorse developmental/cognitive as well as sociocultural, discursive and anthropological perspectives) *has occurred more tellingly*.

In the earlier conferences, there is a proliferation of well-known binaries in the way students' mathematical learning is described and explored, such as: concept image/concept definition construct (Tall and Vinner 1981), procedural/conceptual understanding (Hiebert 1986), process/object (Sfard 1991) and informal/formal modes of reasoning (Fischbein, 1994), often along with cognitive frameworks such as APOS theory (Asiala, Cottrill, Dubinsky, and Schwingendorf 1997). These studies are typically conducted in the context of a relatively limited set of mathematical topics.

Typical investigations and findings in Calculus or Analysis (e.g. Hähkiöniemi 2005) concern students' procedural or conceptual knowledge about the limiting process, their various representations (internal or external) and their fluency in moving across these representations. In a similar spirit, investigations into elementary components in Linear Algebra, such as spaces and subspaces of systems of linear equations (e.g. Trigueros, Oktaç and Manzanero, 2007), highlight what impedes or has the capacity to assist students' development of schemas, such as on variables, sets, functions, equality and vector space. APOS-based genetic decompositions are often proposed in these studies as potentially supportive of this development. With regard to proof and proving, the main theme is what Inglis and Simpson (2005) identify as the two parts of dual process theory (intuition, formalism/abstraction). Students are frequently uneasy with the latter and uncertain about the validity of the former. In these studies, there is also a tendency to juxtapose novice and expert approaches and to explore ways in which novices can learn to emulate experts. One way, explored in several studies, is to assist students' towards constructing rich and meaningful example spaces. For example, Meehan (2007) proposes a systematic sequence of example generation activities in an Analysis course: these activities gradually add extra conditions, alert students to the significance of each condition, and ask students to continuously review and validate their responses.

While the aforementioned binary and stage theories remain part of the vernacular in the more recent studies (Breen, Larson, O'Shea and Pettersson, 2015), they are also being progressively somewhat replaced by more dynamic and fluid theoretical constructs such as the RBC Model (Recognizing, Building with and Constructing: Tabach, Rasmussen, Hershkowitz, and Dreyfus 2015) and constructs originating in discourse analysis, such as those from the theory of commognition (Sfard 2008; Pettersson, Stadler and Tambour, 2013).

Finally, the number of studies with a self-proclaimed *exclusive* focus on the student experience, has decreased over the years (for example, from eight of the twelve papers accepted for publication in CERME4 to also eight, but out of forty-five, papers accepted for publication in CERME9). While we see the distinction between student-centred and teacher-centred studies as slightly artificial, it is at times necessary for pragmatic reasons – as the structure of this part of our chapter suggests. However, since the early days of CERME, there have been some alerts to the importance of focusing on the student-teacher, student-resource, and student-institution interfaces (Cazes, Gueudet, Hersant and Vandebrouck, 2005). The number and quality of studies presented in the more recent conferences on these interfaces is promising in its coverage of a range of pedagogical (Sikko and Pepin 2013), institutional (Liebendörfer and Hochmuth 2015), social (Bergsten and Jablonka 2013) and affective (Stadler 2011) influences on the student experience.

For example, Sikko and Pepin (2013) present a survey of university students in their second year, which explored the teaching and study methods from their first year that the students found most effective. They report the proliferation of active and collaborative ways of working in the student responses, and a limited appreciation for lectures. Farah (2015) focuses on students from preparatory classes for French business schools and especially their "autonomous study and the gestures involved in it" (p. 2097). The study combines qualitative and quantitative methods for answering questions concerning the evolution of individual activities in terms of quantity and forms of study, differences between 'good' and 'weak' students, and in particular the promotion of social relationships established between the students and built with the teachers.

Liebendörfer and Hochmuth (2015) identify different factors which support or hinder the autonomy of first year students, and observe that student teachers are not convinced about the need of university mathematics for teaching at school – or find the transition from school to university mathematics as an often perplexing re-visiting of content and ways of working that seem, simultaneously, familiar and novel (see also Stadler 2011). Retaining a theoretical perspective which networks Anthropological Theory of the Didactic and the construct of didactic contract from the Theory of Didactic Situations, González-Martín (2013) studies how textbooks and teachers' practices shape the institutional didactic contract and in particular its rules about specific parts of the mathematical content; also, he presents cases where students, confronted with tasks which do not obey these rules, may produce inappropriate answers. In these studies, student experience appears as deeply entangled with the pedagogies, the content and the study conditions this experience comes with.

Another way in which research on student experience has changed over the years is the increasing emphasis on teaching of mathematics to non-mathematics specialists. We turn to these developments next.

2.5 UME for non-mathematics specialists

RUME recently takes increased interest in the teaching and learning of mathematics in study programs of other disciplines (sometimes called "non-math majors"). This is compatible with the importance and challenges of mathematics courses in those programs, and the large number of students enrolled. Most papers problematize in various ways the isolation of mathematics courses within such programs and, the increased focus on connectedness of curricula is related to the already mentioned shift from individual to institutional aspects of teaching and learning.

Following this line, Barquero, Bosch and Gascòn (2011) investigated institutional restrictions on teaching modelling in first year "mathematics for natural science" courses at ten different Spanish universities. They identified a dominant epistemology, 'applicationism', concerning the role attributed to mathematics, which is characterized by "a clear separation between mathematics and natural sciences". Together with other restrictions, applicationism makes it difficult for students to relate mathematics to their major study courses like biology, geology, chemistry or environmental sciences. Hernandes Gomes and González-Martin (2015) observed similar beliefs and attitudes and asked how they are possibly influenced by the academic background of lecturers.

That students experience mathematics and their non-mathematical major courses as two separate worlds, is consistent with observations in (Bergsten and Jablonka, 2013), where interview data from students of five different engineering master programs at two Swedish universities are analyzed. Students mostly attribute mathematics to aims like "enhancing individual problem solving abilities" (p. 2294), and not to skills needed in their major study courses. Drawing on Bourdieu (1983), the authors interpret the potential gains from studying mathematics which undergraduate engineering students see, as a form of 'cultural capital'.

At the level of tasks, the separation in two worlds and how it is represented in modelling-cycles has been questioned by Biehler et al. (2015). A construed student-expert solution for a task from a German first year course on "Foundations of Electrical Engineering" exemplarily shows that "a division into separate phases […] is not adequate" (p. 2066), as the electrical engineering task requires a 'non-separated' knowledge. Similarly, Xhonneux and Henry (2011) conducted a praxeological analysis of how Lagrange's multiplier method is presented in mathematics and economics textbooks, showing that in mathematics, one begins with Lagrange's Theorem and its justification, which is then applied to particular cases - whereas in economics, students encounter the technique of Lagrange's multipliers as an integrated part of solving economic optimization problems.

3. Mathematical Education at University: what could it be?

Proposing and discussing possibilities for improving the teaching of university mathematics are important tasks for RUME. We now turn to examples of CERME research presenting experiments and interventions, focused on developing UME practices.

3.1. Research on and for innovation in UME

Epistemologically oriented research is a necessary prerequisite for preparing concrete interventions on a scientific basis. So far only a few papers actually describe research driven interventions, with a systematic evaluation of the implemented measures. Most studies present explorative, small-scale experiments, which are of a more preliminary nature.

Hausberger (2015), for example, studied the case of a course on abstract Algebra for university students with a background in group theory in their third year. He developed a series of activities regarding so called "banquets", starting with a logical investigation of their underlying axiomatic system and the classification of adequate models, following by "the elaboration of an abstract theory of tables and structure theorem for banquets" (p. 2148) and finally establishing a link to permutations. This proposal is rooted in the French methodology of didactic engineering, and is based on a complex net of frameworks concerning abstraction, semiotics, relations between syntax and semantics, and algebraic structuralism.

Kondratieva (2015) experimented “interconnecting problems” to provide students with more knowledge about how the basic mathematical disciplines such as geometry and calculus are linked; revisiting and linking knowledge from earlier courses was found to be a promising direction for engaging students in more independent problem solving.

Other studies concern interventions at larger scale, involving courses or workshops designed to support or prepare students for their very first courses at university. Typical examples are the VEMA project, involving several German universities (Biehler, Fischer, Hochmuth and Wasson, 2011), and the “springboard operation” which focuses on linear algebra at a Belgian university (De Vleschouwer and Gueudet, 2011).

For RUME, the teaching and supervision of graduate and in particular post-graduate students in mathematics education is of specific importance, as many Ph.D. students who have graduated in mathematics need to ‘transit’ from a science to a social sciences paradigm. This issue is taken up in (Nardi, 2013), with an intervention based on principles like participation, cultural sensitivity, creativity and critical thinking and addresses in particular the issues ‘engaging with research literature’, ‘forming the conceptual and theoretical framework of a research project’ and ‘choosing and applying data analysis methods’.

In her CERME 5 plenary, Artigue (2007) combines general considerations about educational inventions based on digital technologies with a discussion of theoretical issues concerning RME. She argues that “research dealing with digital technologies reflects the general trends and major evolutions of the field but it is also a source of inspiration for these” (p. 69). The dynamics of creativity and evolution in this field affects the balance between educational systems and society, hence makes inherent tensions visible and thereby, potentially, scientifically negotiable. These general observations also apply to UME. But so far, CERME has seen only a few studies on experimental uses of digital technologies in UME.

For instance, Dreyfus, Hillel and Sierpinska (1999) proposed and analysed a teaching of Linear Algebra using digital geometry software (Cabri). Testing it with undergraduate students, they evidenced that the choices made in the software, for example for the representation of vectors, strongly influenced students’ conceptions.

Cazes, Gueudet, Hersant and Vandebrouck (2005) studied the introduction of web resources in two different French universities with a focus on supporting the development of problem-solving competences. Based on Schoenfeld’s (1985) appraisal of problem-solving activities as important for the learning of mathematics, the authors provided online exercise collections and investigated if the “software allow the students to develop a real problem solving activity” (Cazes et al., 2005, p. 1745). For a minor group of students this could obviously be affirmed, but the majority of students engaged in manifold activities without succeeding with the basic problem. The authors indicate the “need for further studies to produce more precise descriptions of the students’ activity” (p. 1745).

Based on ideas from ATD, in particular the dialectics between ‘media’ and ‘milieu”, Grønbæk and Winsløw (2015) designed an interactive self-study module on complex numbers for non-mathematics students in an introductory mathematics course. The authors explicitly questioned the extent students actually “develop a critical and autonomous relationship to the ‘answers’ found in the media” (p. 2133). According to this aim, the authors intended “to create a media-milieu dialectics in three ways: as a generator of dynamic text (media with embedded milieu), as a generator of drills for techniques to solve specific tasks with feedback to students’ solution proposals (milieu with embedded media), and to explore and exhibit phenomena (media with embedded milieu)” (p. 2134). Whereas students criticized the lack of feedback in an anonymous on-line evaluation, there are indicators that they appreciated the idea of drill.

In general, studies and above all evaluations of interventions show the need for specific instruments that allow to capture and to analyze students' activities more accurately (cf. Section 2.3).

3.2. Professionalization of UME practice

As witnessed by other chapters in this book, RME has increasingly turned its attention to mathematics teachers, to investigate their practices, beliefs, knowledge and education, but also to involve them as partners in research - not least those directed towards innovation and development. That the interest in teachers' practices and beliefs has also materialized in RUME was exemplified in Section 2.2. Much less research exists when it comes to university teachers' knowledge and its development through formalized education. A theoretical model was proposed by Winsløw (2005), in terms of mathematical and didactical praxeologies of university mathematicians, to investigate the practice and knowledge which these deploy in research and teaching, and the possible relations between them.

University teachers are often "experts" in a field they do research on, and this may be a reason why their knowledge is not much investigated, let alone questioned. Studies of university teachers' knowledge of specific teaching methods (Mesa and Cawley, 2015) and of their practices in problematic or "new" situations, such as in dealing with digital resources (Gueudet, 2013), contribute to raise the question of their professional development as *teachers*, and the contribution RUME may supply. Matthews and Jaworsky (2011) report on experiences from a seminar for "mathematicians and mathematics educators", which began as a voluntary and explorative activity, and has since become part of the professional development program for new mathematics lecturers at the authors' university.

4. The future of UME and RUME

Studies of the current practices and results of UME (Section 2) show that this field of practice is faced with a number of considerable challenges. At CERME as well as in RUME more generally, we still have mostly relatively local knowledge of these challenges, based on small scale studies; the same goes for studies based on developmental and experimental "intervention" (Section 3.1). International collaboration, as increasingly fostered and appearing in CERME, should help to establish our knowledge of the challenges, as well as the viability of possible solutions, on a firmer and more essence-oriented basis - and also to further develop the precision and sophistication of theoretical models and empirical methodology behind the research.

Ultimately, the motivation (and funding!) of RUME is largely connected to the purpose of *solving (some) problems faced by UME.* As in pre-university levels, this naturally brings the teachers in focus, along with the questions of professional development and formal university teacher education (Section 3.2), which we foresee will gain importance both in research questions and as rationale for RUME.

At the same time there is an increasing realization of the importance of teachers as not only "audiences" but also "partners" in efforts to promote change. It should be noted that in RUME, one cannot talk of "teachers" and "researchers" without observing that many university mathematics teachers are in fact researchers, although not typically with RUME as their research specialty. Moreover, in the CERME papers reviewed for this chapter, especially those from the last ten years, we find still more authors and coauthors who, besides (as such) being involved in RUME, are also teachers of mathematics at university, frequently in the contexts their study investigates. This also means that the RUME group at CERME - and conceivably RUME more generally - include scholars with a variety of backgrounds and experience in research, including pure and applied mathematics. For a number of authors with their PhD in mathematics, including several of those quoted in this chapter, CERME has functioned as a venue for learning on and induction into RUME - not least thanks to its focus on international *collaboration* and constructive *communication*. Such interfaces between teachers and RUME can be conceived as especially important to create and sustain research based innovations in UME. As mentioned in Section 3.1, these currently tend to be quite local and small scale, in the sense of involving just one course (or part of it) at one university. In the future, we hope and expect to see more RUME studies which stretch across institutions, courses and even national borders, and CERME should be a promising framework to enable this. We also believe that such developments will help to make results and projects from RUME more attractive and useful to teachers with their academic core in mathematics or other sciences.

This, finally, raises the question of *impact* of RUME on UME. Besides collaboration and communication with teachers through professional development and research, we should mention also the potential, but currently quite limited, impact at the level of innovation of curricula and policy. Some of the more global problems identified by RUME, such as transition problems between secondary and tertiary institutions (Section 2.3), the isolation of mathematics courses in non-mathematics majors (Section 2.5) or the need for more connectivity across courses within study programs (Sections 2.5 and 3.1), clearly call for research and impact at this level.

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