Early Years Mathematics

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1. Introduction

The Early Years Mathematics Working (EYM) group met for the first time in 2009. As such, it is a relatively "young" group, vibrant, and still growing. Perhaps the first question that must be dealt with at the onset is the necessity of this working group, or in other words, what makes early years mathematics special that it warrants its own working group. Thus, this chapter begins with a discussion of what it means to develop young children's mathematics knowledge and the importance of learning mathematics during the early years. The second part of this chapter deals with themes which have continuously interested researchers in this group, as well as ideas which have been developed over the last four CERME conferences. The third part of this chapter takes a look at connections between this working group and other CERME working groups. Finally, the last part reviews some of the challenges this group faces as well as future research directions.

2. Mathematics for young children

In their review of early childhood mathematics learning, Clements and Sarama (2007) noted that "researchers have changed from a position that young children have little or no knowledge of or capacity to learn mathematics…to theories that posit competencies that are either innate or develop in the first years of life" (p. 462). But, what is the mathematics that children are learning at such a young age? And what age is considered young?

Historically, the construct of "early years" has been associated with informal learning prior to formal schooling. However, current curricula and standards for preschool mathematics (e.g., in England the Curriculum Guidance for the Foundation Stage (CGFSm DfEE/QCA, 2000)), offer several suggestions for practitioners in how to specifically foster children's knowledge of counting, calculations, shapes, and measures. In addition, several curricula also advocate the promotion of mathematical processes such as problem solving, reasoning, and justifying conjectures (e.g., Israel National Preschool Mathematics Curriculum [INPMC], 2008, p. 8). In the EYM group at CERME, we have discussed children's development of mathematical concepts (e.g., Koleza & Giannisi, 2013), as well as how to encourage children's ability to explain and justify their reasoning, and promote early generalization and abstraction (e.g., Vighi, 2013).

Regarding the age of "young" learners, we refer to children between the ages of 3 and 8 years.[1] This is in line with the inclusive spirit of CERME and takes into account the different transition ages between preschool and primary school in different countries. It also fosters cooperation between mathematics education researchers of different age groups. At the 2015 CERME conference, there were nine papers which reported on studies of primary school children (up to age 8). Although those papers may have also fit in with other groups (e.g., the Arithmetic and Number Systems group), the approach in the EYM group is to emphasize the development over time of early mathematical concepts, focusing on the transition between informal and more formal learning environments. (See also Section 3 of this chapter.)

There are several important reasons for focusing on EYM. First, focusing on the transition to formal schooling, not all children come to first grade with the same knowledge. For example, low-income children often come to first grade with less mathematical knowledge than higher-income children (Starkey & Klein, 2000). Tsamir et al. (2011) found significant differences between the knowledge of abused and neglected children and other children. In addition, research has found that early knowledge of mathematics may be seen as a predictor of later school success (Duncan, et al., 2007). Finally, when the EYM group convened for the first time, the introduction noted how several countries, including Germany, Finland, Cyprus, Denmark, and others, have become increasingly aware of the need to support mathematics learning during the early years. Yet, not all countries have a mandatory or even recommended curriculum for this age, nor do all have compulsory or financially supported education for young children. By supporting the work of the EYM group, the mathematics education community is sending a message that this research is vital and can be used to inform countries who are developing curricula for this age. In the next section, we highlight a few themes which have continuously run through CERME meetings at the EYM group.

3. Sustained themes

There are at least three particularly important issues related to EYM education. The first is the importance of what the child learns at home, before or alongside formal instructions at school. At

[1] Preschool in many countries (e.g. Sweden) is from 1-5 but our concern is for the ages 3-5. Some countries label the first years as kindergarten, and some label kindergarten as the year prior to primary school. In this chapter, we use preschool and kindergarten almost interchangeably (mainly in the way the authors we refer to use them).

times, children learn mathematical concepts at home that are not necessarily utilized in the most appropriate way (Meaney, 2009). In Germany, studies have investigated mathematical learning embedded in family discourse and how family members can support mathematical learning (e.g., Brandt & Tiedemann, 2009).

From the adults at home, we turn towards the adults in the classroom, namely the teachers. At the first EYM meeting, one paper dealt with preschool teachers' mathematics beliefs (Benz, 2009) and showed that although teachers may agree with a constructivist approach to learning, they may agree more with an acquisitionist approach. The author suggested that professional development could help teachers in supporting children's own constructions. It was also shown that kindergarten teachers' views of learning and teaching mathematics differ from primary school teachers' beliefs, due, in part, to different curricula as well as different trainings (Schuler, Kramer, Kröger, & Wittmann, 2013). For these and other reasons, several countries have initiated professional development programs for practicing preschool teachers. In Italy, for example, teacher educators, education committees, policy makers, and teachers combined their efforts to help teachers implement a new mathematics curriculum for preschool children (Bartolini Bussi, 2013). At the last CERME, studies dealt with the roles of the preschool teacher and their views of how to guide children's learning (e.g., Delacoeur, 2015).

Perhaps the most discussed and debated issue in the early mathematics education community regards the roles of instruction and formal learning versus the roles of play and types of play in early years mathematics (e.g., Aplaslan & Erden, 2015). We face differences linked to our cultures and the roles of kindergarten and early schooling. According to the OECD (2006), kindergartens in the Nordic and Central European countries are situated within a social pedagogical tradition, as an educational institution where upbringing, care, play and learning are the core enterprises and where free play has a key role. The OECD mentions countries in the Western part of Europe (UK, Ireland, France and the Netherlands) as having a pre-primary approach, meaning they "tend to introduce the contents and methods of primary schooling into early education" (p. 61). Thus, kindergartens in the Western part of Europe, compared to Nordic and Central Europe have stronger similarities with formal schooling and less focus on play.

The notion and interpretation of play are problematic and vary between contexts, countries and theoretical frameworks. Learning can be difficult to capture in play-based activities compared to

a more structured classic school type of activity, where more is expressed orally than in written form. Thus, there is also a need to discuss methodological perspectives on data collection and data analysis of play situations and exploration (Vogel & Jung, 2013). In this chapter we add the term *playful learning*. This term takes into account that for a child, play and learning are one and the same. Playful learning builds on developmental research which has documented children of all ages experimenting with mathematical concepts through play (Hirsh-Pasek, Golinkoff, Berk, & Singer, 2009). In their outline of playful learning, Hirsh-Pasek et al. distinguish between *free play* and *guided play*. In guided play, a teacher typically offers activities and materials for the children to engage with, where the aim is to learn some mathematical concept. Several studies explored characteristics of guided play such as when children and parents play board games together (Tubach, 2015), or where children play with games designed to develop children's conceptions of number (e.g., Sinclair & SedaghatJou, 2013). In these guided play situations, the teacher typically considers opportunities to interact in ways that support children's learning of these concepts. Free play, on the other hand, takes part without any input or structure from the teachers and, with the exception of Flottorp's (2011) study, which captured two boys' verbal and non-verbal expressions in an outside free play situation, has rarely been studied in our group.

An issue related to the balance between children's free play, guided play and the role of formal instruction and teaching materials, is the notion of agency. In free play situations, children´s own choices and explorations take part in their natural setting, and the mathematics may be hidden rather than explicit. In guided play, children work in a more structured way supported by materials, where the mathematics is made visible and the "play" is structured by the teacher. In the latter situation, children may become passive "receivers" of ready-made structures. In a free play situation, the children have the agency; in a guided play situation, the teacher and the material being offered have the main agency and children's agency is limited to a few options (e.g., Erfjord, Carlsen, & Hundeland, 2015). The learner needs to have some agency in order to learn. However, full agency may be problematic from a mathematical point of view where concepts are crucial and build on each other.

4. The relevance of EYM research to other research groups

The transition from preschool to primary school is another concern of the EYM group. Mathematical concepts develop over time, and although other working groups at CERME

discuss arithmetic, geometry, and algebraic thinking, these conceptualizations begin before primary school and as such, are also discussed within the EYM group.

Alpaslan and Erden (2015) presented a review of EYM research papers from 2000 to 2013 published in seven peer-reviewed research journals. The research topic most investigated was number systems and arithmetic, a topic addressed in many EYM papers at CERME. We are also aware of the fact that several papers in the Arithmetic and Number Systems group study this topic among children ages 6-8 years old. The importance of these papers to the EYM group is seeing how numerical concepts develop over time, from a very young age to the first years of school. Thus, although number concepts are dealt with in the Arithmetic group, we address these concepts in the EYM group, not just from a pre-school point of view, but also with a look towards the first years of schooling, addressing transitions within educational systems (e.g. Vennberg, 2015).

One of those transitions includes developing a sense for arithmetic operations by building on key number concepts. For example, Sinclair and SedaghatJou (2013) showed how a touchscreen application may engage children with the concept of cardinality, which can then lead them to use cardinality when joining (i.e., adding) two groups. In another study, Maffia and Mariotti (2015) described how second graders worked with an artefact to learn about the distributive property. Notable was the movement from personal meanings, to mathematical meanings, a movement which is of particular interest to the EYM group, where it can be adapted to other mathematical properties. Rational number concepts, usually developed during the elementary school years, may have their roots in early fraction concepts such as how children conceptualize the notion of a half (Tirosh, Tsamir, Tabach, Levenson, & Barkai, 2011).

Another content strand discussed at the EYM group is geometrical thinking, which is the main theme of the Geometrical Thinking group. The difference between the groups is that at the preschool level, geometrical learning is mostly related to shapes, whereas at the upper levels, geometrical thinking encompasses a wider perspective including transformations, topology, and proof schemes. This does not mean that some overlap does not occur. In 2015, a paper was presented at the Geometrical Thinking group regarding six-year olds students' knowledge of intuitive triangles (Rodrigues & Serrazina, 2015). This paper would have been interesting for the EYM group and in fact, Rodrigues and Serrazina (2015) suggested that some of their findings

might be due to students' previous experiences with shapes. Those authors, as well as the participants of the EYM group, might have benefitted from the presentation of that paper to the EYM group. In the opposite direction, a geometry-related paper presented at the EYM group dealt with defining rectangles and squares in first grade (Bartolini Bussi & Baccaglini-Frank, 2015). This paper exemplified an EYM approach to geometrical definition by taking into consideration young children's natural tendency to place these figures into distinct categories, a tendency reinforced by everyday language.

A more recent development within our group is the integration of Information and Communication Technologies (ICT) with preschool mathematics, connecting to the "Technologies and Resources" group in mathematics education (TWG15). The issue of ICT first appeared in the EYM group in 2011 (Ladel & Kortenkamp, 2011) investigating the linking of different forms of representation (hands, fingers, and multi-touch technologies). This area was futher explored in 2013. Once again, cultural differences were noted along with the previously mentioned quandrum between free play and directed learning in preschool. In Sweden (Lange & Meaney, 2013), one study explored the mathematics which may be hidden in popular (not necessarily mathematical) games played on the iPad. On the other hand, Sinclair and Sedaghatjou (2013), from Canada, investigated a game (Touch Counts) specifically designed to promote young children's knowledge of number concepts. Hundeland, Erfjord and Carlsen (2013) explored the roles of the teacher when employing ICT related mathematics activities and how ICT can support mathematical learning processes through exploration and discovery of mathematical relations. This area of research led to additional research questions, such as the difference between engaging children with concrete versus virtual manipulatives.

In 2015, a short activity with Touch Counts was analysed, focusing on the audible, the visible, and the tangible (Pimm & Sinclair, 2015) seeking to gain insight into the nature of number, in particular the ordinal aspect, in this complex assemblage. Demetriou (2015) compared the use of concrete and virtual manipulatives in three symmetry tasks. Bartolini Bussi & Baccaglini-Frank (2015) presented fragments of a first grade experiment where the seeds were sown for a mathematical definition of rectangles that includes squares, by means of programming a very simple robot. It is important to note that all ICT related papers in our group investigated the potential use of ICT in its own right, and not merely as a replacement for concrete manipulatives. We expect this to be a growing concern for our group.

5. Looking back and looking ahead – challenges and future research directions

5.1 Challenges

From the first meeting of the EYM group, scholars from different countries brought with them research traditions and related theoretical frameworks that appeared to be strictly linked to cultural contexts. At CERME 8, there was a growing interest in exploring differences among the various cultures, sparked by two papers, one from China (Sun, 2013) and one from Italy (Ramploud & Di Paola, 2013). Within the context of addition and subtraction, these papers explored task design in different countries, different epistemologies, pedagogies, and beliefs related to cognitive processes. This issue of different research traditions was raised again in CERME 9, where the presence of several theoretical traditions was noted in the introduction to this group's papers:

"In most cases, ... theoretical frameworks are chosen with reference to ... authors from the same country; this choice could be misunderstood as patriotism, but it is not necessary the case. For instance, the semiotic mediation theory is useful in countries where the focus is on long term studies, which in turn depends on the institutional role of a teacher working for more than one year with the same group of pupils […] The relationship between contexts and theoretical frameworks is a challenge for the diffusion of findings and the possibility of exploiting findings from different cultural contexts […].." (p. 1886-1889)

5.2 Future directions

The EYM working group faces many difficult, but important and interesting questions. In Section 3, we discussed balancing agency, free play, guided play, structured materials, and teacher guidance. Some of our differences are contextual and not something we, as researchers can influence directly. As previously noted, in northern Europe there is a strong emphasis on play whereas in the Western part of Europe there is a tendency for more structured mathematical activities. Bringing together these cultures at CERME allows Northern European researchers to explore more structured activities and for Western European researchers to explore more play based activities. Exploring together play and learning from a developmental and research perceptive could link members of EYM at CERME. The question of how to approach the "education" of children was a key question at CERME in 2011. One concrete result of this issue was a decision by participants in Germany to organize a workshop-based conference during the

spring of 2012. This was the start of POEM – A Mathematics Education Perspective on Early Mathematics Learning between the Poles of Instruction and Construction. Participants from the EYM-group at CERME, as well as others, met, and have continued to meet every second spring (in between the CERME congresses). The overall focus for POEM has been the seemingly distant poles of instruction and construction, with some arguing that children's play should be the starting point, and playful activities should contain some elements of instruction (e.g., van Oers, 2014). Another conference for EYM is the Children's Mathematical Education conference (CME) taking place in Poland every second year. We believe strong links between these conferences strengthens each conference and group. At CERME 10, the EYM group will have group leaders centrally engaged in POEM and CME.

Although previous researchers (e.g., Clements & Sarama, 2007) outlined key early conceptions of number and geometry, we believe that additional research is necessary. Such research could help illuminate for practitioners ways in which to develop rich mathematical concepts for children in different settings, using different kinds of activities, and tools (manipulative as well as virtual). As researchers, we consider both where we stand and where we wish to go. As such, we believe that at future EYM working group sessions, it would be mutually beneficial for researchers from different countries to discuss new avenues of research and questions. We end by giving an example: We have seen several CERME research papers (e.g., Sæbbe & Mosvold, 2015) that have taken the teacher's perspective and studied, for example, the role of questions in mathematical activities in EYM. However, there is little research into the nature and task design of mathematical activities and teacher's orchestration that may foster children's questioning and children's own investigations. This can be a next step.

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